Name:____

Professors name:____

Due date: 12/11

1. Solve the following initial value problem:

$$\frac{dw}{dx} = x\sqrt{w}, \qquad w(0) = 4.$$

2. An object in free fall may be modeled by assuming that the only forces at work are the gravitational force and air resistance, which is proportional to the speed of the object and acts in the opposite direction of the fall. Thus, the differential equation that models free fall is:

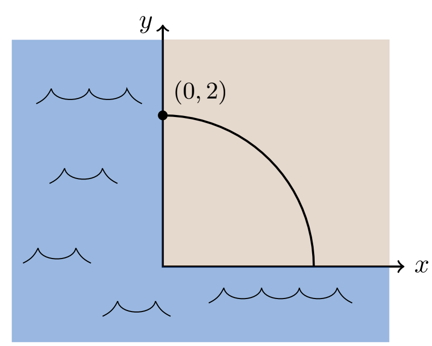
$$m\frac{d^2x}{dt^2} = mg - m\mu\frac{dx}{dt}$$

where m is the mass of the object and μ is an air resistance constant.

Solve for the position of the object at any given time, $x(t)$, assuming the initial height is at 10 meters and the object is released from rest.

3. A mathematical problem of antiquity is the Dido problem, dating back to 825 BCE. Queen Dido fled violence in her homeland for what is the modern city of Tunis, and negotiated with the locals for "as much land as could be bound by a bull's hide." Dido needed to maximize the area enclosed by a fixed perimeter: this is the orginal *isoperimetric problem.*

   (a) Find parametric equations $x = h(t), y = g(t)$ which describe the quarter circle of radius 2 centered at the origin with $0 \leq t \leq 1$ and initial point $(f(0), g(0)) = (0, 2)$.
   (b) What is the tangent line at $(\sqrt{2}, \sqrt{2})$?
   (c) Compute the arclength of the parametric equations you found in Part (a).
   (d) **Bonus point**: Try instead to draw a line starting at (0,2) which meets the x-axis with length equal to what you found in Part (c). Compare the areas enclosed with these two shapes: which one should Dido draw?



4. A circular corral of radius 3 is enclosed by a fence. A goat inside the corral is tied to the fence with a rope of length a, where $0 \leq a \leq 6$. What is the area of the region inside the corral that the goat can graze? Verify your answer for $a = 0$ and $a = 6$.