## NAME: ______________________________

**Read all of the following information before starting the exam:**

- **WRITE YOUR NAME AT THE TOP OF EACH PAGE (you will lose points otherwise)**
- **DO NOT WRITE ON THE FRONT OR BACK OF THE FIRST PAGE other that writing your name.**
- **Show all work and give explanations where needed. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).**
- **Use only the paper provided, your one page notes and a pen or pencil.**
- **Write you answer in the box provided.**
- **This test has 5 problems. and is worth 50 points, It is your responsibility to make sure that you have all of the pages!**
- **Good luck!**

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| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| total | |

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**1.** (*16 points*) Let $f(x) = \sqrt{x}$

a) Give the linear approximation for $f(x)$ centered at $x = 1$ and use the linear approximation to estimate $\sqrt{2}$

b) Give the degree three Taylor polynomial for for $f(x)$ centered at $x = 1$ and use this Taylor polynomial to estimate $\sqrt{2}$

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c) Give an expression (in terms of $x$) for an upper bound for $|R_3(x)|$ assume $x > 1$. Then use this to give an upper bound for the error in your estimate for $\sqrt{2}$ in the previous part (part b.)

d) Based on parts b) and c) give values $m$ and $M$ for which you are sure $m \leq \sqrt{2} \leq M$.

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**2.** (*6 points*) Give the interval of convergence for the power series $\sum_{k=0}^{\infty} \frac{(-1)^k x^{k+1}}{k+1}$.

**3.** (*8 points*) Use the infinite Riemann sum (the Riemann sum where you let the number of estimating rectangles go to infinity) to give the area under $f(x) = x^2$ on the interval $(0, k)$ where $k$ is some number. Your answer should be in terms of $k$. Then using your previous work give the area under $f(x)$ on the following three intervals, $(0, 1)$, $(0, 2)$ and $(0, 3)$. You may use integrals to check your work, but you will be graded on this as a Riemann sum problem. Here is a helpfull formuula $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

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**4.** (*8 points*) **Choose two of the following.** Make it clear which you choose.

- A random number generator will build a sequence in the following way. A random number $a_1$ will be chosen so that $0 < a_1 < \frac{1}{2}$, from then on $a_{n+1}$ will be chosen so that $a_n < a_{n+1} < 1 - \frac{1}{n}$ Answer the following with short explanations:

  1. Is the sequence monotonic (if so is it increasing or decreasing?)
  2. Is the sequence bounded (if so give the bound?)
  3. Does the sequence converge (if so why and can you say to what?)

- A person will take a 100mg of a drug with half-life 5 hours every 5 hours. Give a recursive sequence $\{d_n\}$ where $d_n$ is the amount of drug in the persons system immediately after the $n$-th dose. For example $d_1$ is the amount of drug in the persons system immediately after the first dose, and $d_2$ is the amount of drug in the persons system immediately after the second dose. Assume $\lim_{n\to\infty} d_n = L$ where $L$ is a number. Give the value of $L$ and an interpretation of what $L$ means.

- Determine if the following sequence converges, and if so to what.

$$a_k = \left(1 - \frac{3}{k}\right)^{k^2}$$

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**5.** (*12 points*) **Choose 4 of the following.** Make it clear which you choose. Apply one of the convergence tests taught in class to determine if the series converges. You must state the test you are using, and show all work required for the test. If you state (correctly) what the series converges to you will get a bonus point, but if you give the incorrect limit you will lose a point.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{10^n}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n} - \frac{1}{3^{n+1}}$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}2^{3n}}{4^{2n+1}}$

d) $\displaystyle\sum_{n=3}^{\infty} \frac{k^2}{k^5 + \cos(k)}$

e) $\displaystyle\sum_{n=3}^{\infty} \left(\frac{[\ln(k)]^2}{k}\right)^k$