Math 36	Tufts University	Monday, May 4th 2015
Applied Calculus 2	Department of Mathematics	3:30-5:30PM
	Final exam	

NAME: ______________________________ **Instructor: Zachary Faubion**

Read all of the following information before starting the exam:

- **WRITE YOUR NAME AT THE TOP OF EACH PAGE (you will lose points otherwise.)**
- **DO NOT WRITE ON THE FRONT OR BACK OF THE FIRST PAGE other than writing your name.**
- **Show all work and give explanations where needed. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).**
- **Use only the paper provided, your one page notes and a pen or pencil.**
- **Box your answers if possible.**
- **This test has 8 problems. and is worth 60 points, It is your responsibility to make sure that you have all of the pages!**
- **Good luck!**

1	
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total	

NAME:

1. (*5 points*) Evaluate $\int_0^1 x\ln(x)\ dx$. Note this is an improper integral. You must show all work (show the computations involved in computing the integral and limits.)

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2. (*5 points*) Choose one of the following 2 problems, make it clear which you choose.

a) Compute the limit of $\sum_{k=2}^{1000} \frac{1}{k^2 + 7k + 12}$.

b) Compute the arc length of $f(x) = \frac{e^x + e^{-x}}{2}$ from 0 to ln(2).

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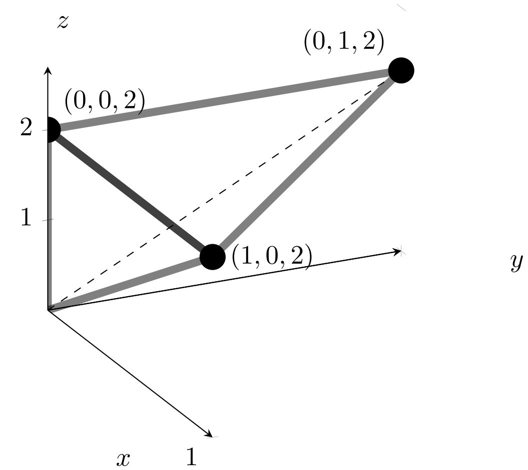
3. (*8 points*) Find the work required to empty the tank described below (lift the water to the top of the tank,) assume the density of the liquid is ρ and use g as the gravity constant. The shape is the tetrahedron with vertex $(0,0,0)$, $(0,0,2)$, $(0,1,2)$ and $(1,0,2)$. Hint, it may help to find the equation of the line in the x,z plane between the points $(0,0,0)$ and $(1,0,2)$ (which should be a linear equation in terms of z and x) and also the equation of the line in the y,z plane between the points $(0,0,0)$ and $(0,1,2)$ (which should be a linear equation in terms of y and z.)

You will be given partial credit so draw and label the estimating shape and set up as much of the integral as you can.



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4. (*12 points*) Compute the following integrals. Show all work.

a) $\int_0^{\pi} \sin^5(x)\cos^3(x)\ dx$

b) $\int x^3 \sin(x^2)\ dx$

c) $\int_0^{2} \sqrt{4-x^2}\ dx$

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5. (*12 points*) **Use a series convergence test** to determine if the following series converge. You must state which test you use and then apply it correctly. A bonus point if you state what the series converges to.

a) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k\ln(k)}$

b) $\displaystyle\sum_{k=1}^{\infty} \sqrt{\frac{k}{k^3+1}}$

c) $\displaystyle\sum_{k=3}^{\infty} \frac{(-1)^k 2^{k+1}}{3^{2k}}$

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6. (*7 points*) Choose 1 of the following, make it clear which you choose.

a) Give a the power series representation for $\arctan(x)$, make sure to give the interval of convergence. Then use this power series to give a power series representation for $\arctan(3x)$, make sure to give the interval of convergence. For a bonus point give a series representation for π.

b) Give the 5th degree Taylor polynomial for $f(x) = \sin(x)$ centered at 0, use the Taylor polynomial to estimate $\sin(1)$ and then give an upper bound for the error of your estimate.

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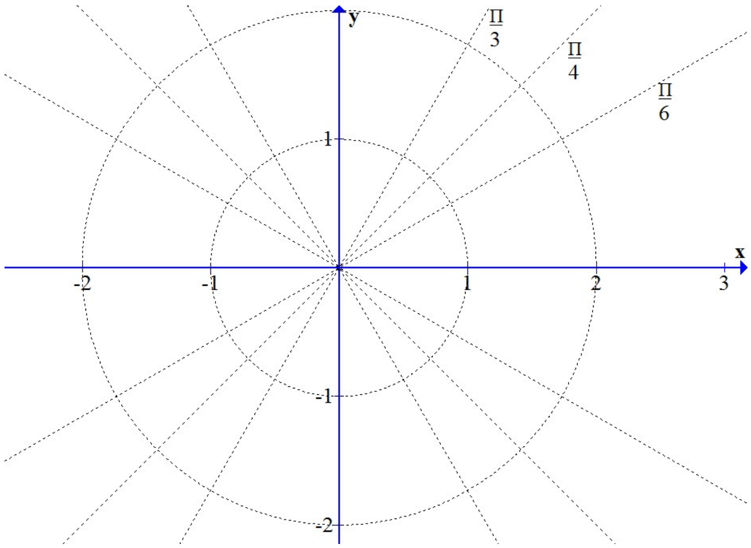
7. (*6 points*) Graph the polar function $r(\theta) = 2\sin(3\theta)$ and on the graph provided. Then set up the integral (but don't integrate) to find the area inside bounded $r(\theta)$, but outside $r(\theta) = \sqrt{2}$.



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8. (*5 points*) Choose 1 of the following, make it clear which you choose.

a) Give the parametric equation for line segment between $(-1, -3)$ and $(2, 15)$, and give the paremetric equation for a circle centered at $(1, 3)$ of radius 2 oriented clockwise. Make sure to give the domain of the equations.

b) Set up the integral for the volume of the shape obtained by rotating the region R about the y-axis. R is the region bounded by $f(x) = \sin(x)$, the x-axis, $x = 0$ and $x = \pi$. This is best done with the shell method.

c) Give the interval of convergence for the power series $\displaystyle\sum_{k=1}^{\infty} \frac{(x-2)^k}{\sqrt{k}}$