NAME: ______________________________

Read all of the following information before starting the exam:

- **WRITE YOUR NAME AT THE TOP OF EACH PAGE (you will lose points otherwise)**
- **DO NOT WRITE ON THE FRONT OR BACK OF THE FIRST PAGE other that writing your name.**
- **Show all work and give explanations where needed. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).**
- **Use only the paper provided, your one page notes and a pen or pencil. If you need additional scratch paper some will be provided.**
- **Write you answer in the box provided.**
- **This test has 10 problems with one bonus problem and is worth 70 points (not counting the bonus problem, It is your responsibility to make sure that you have all of the pages!**
- **Good luck!**

problem	value	points
1	9	
2	6	
3	6	
4	9	
5	8	
6	8	
7	8	
8	6	
9	6	
10	5	
total	70	

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1. (*9 points*) **Choose 3 of the following**. State whether the following infinite series converge or diverge, there is no need to compute their limits.

a) $\sum_{k=0}^{\infty} \sin\left(\frac{k}{k+1}\right)$

b) $\sum_{k=1}^{\infty} \tan\left(\frac{1}{k^2}\right)$

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c) $\sum_{k=0}^{\infty} \frac{k^k}{2^{k!}}$

d) $\sum_{k=0}^{\infty} \frac{1}{k \ln^2(k)}$

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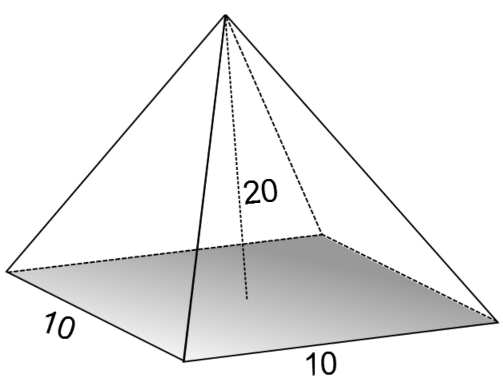
2. (*6 points*) **Choose 2 of the following**. State whether the following infinite series converge or diverge, **it they converge determine their limit**.

a) $\sum_{k=3}^{\infty} (-1)^k \; \frac{2^{(2k+1)}}{16^{(k+1)}}$

b) $\sum_{k=1}^{\infty} \ln\left(\frac{k}{k+1}\right)$

c) $\sum_{k=0}^{201} \frac{1}{k^2+3k+2}$

3. (*6 points*) Find the work required to lift the water out of the top of the tank in the shape of a right square based pyramid with a base length 10 units and a height of 20 units. Use g as the constant for gravity and ρ as the constant for water density. You should ignore all units in this calculation. **Set up the integral but do not evaluate.**



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4. (*9 points*) Let $f(x) = e^{-\frac{x}{2}}$

a) Give the fourth degree Taylor polynomial for $f(x)$ centered at zero.

b) Use the Taylor polynomial from part a) to estimate e.

c) Give an upper bound for the error of your estimate (keep your estimates within reason.)

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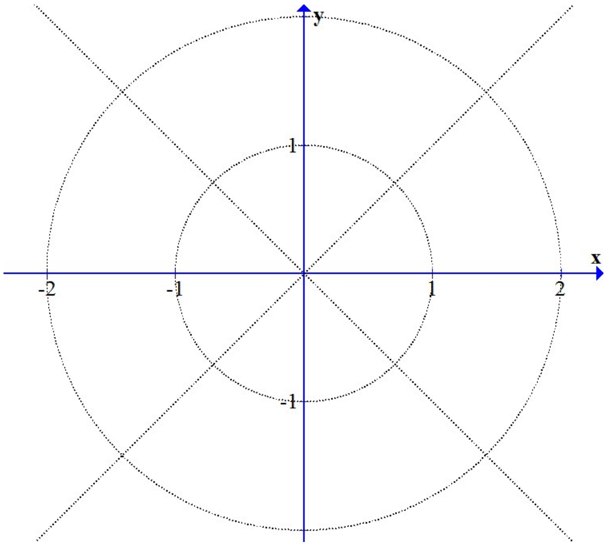
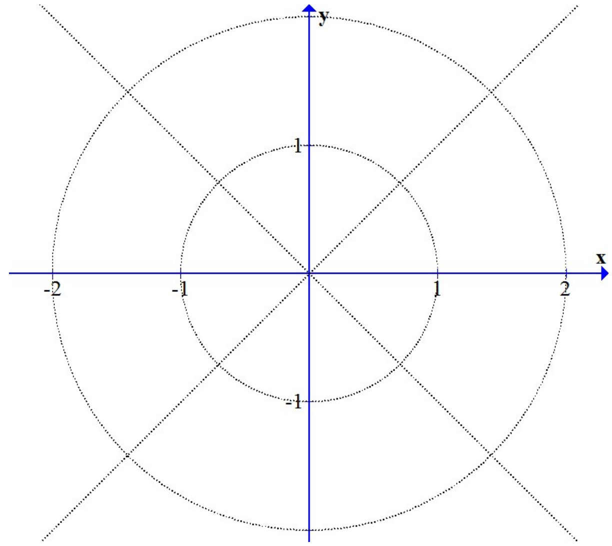
5. (*8 points*) Graph the polar functions $r(\theta) = 2\cos(2t)$ and $r(\theta) = 2\sin(2\theta)$ on the graph below and find the area bounded inside both functions.





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6. (*8 points*) In this problem we will prove that the alternating harmonic series $\sum_{k=1}^{\infty} \frac{(-1)^k}{k}$ converges to $\ln(2)$.

a) Derive a power series representation for $\ln(1-x)$ make sure give the interval of convergence. You must show all work.

b) Use the power series from part a) to find a power series for $\ln(1+x)$, make sure to give the interval of convergence.

c) Use part b) to show that $\ln(2) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k}$. You may have to “fix up” the sum to look right.

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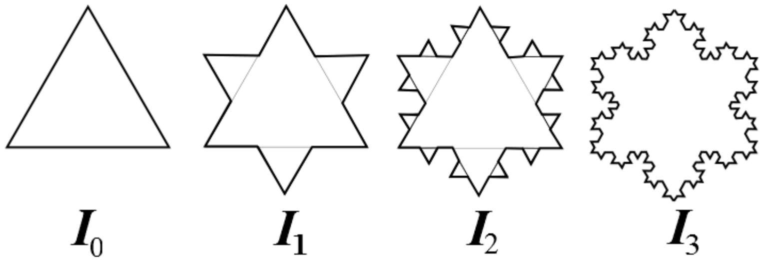
7. (*5 points*) The fractal snowflake is constructed as follows: Let I_0 be an equilateral triangle with sides of length l. The figure I_1 is obtained by replacing the middle third of each side of I_0 by a new outward equilateral triangle with sides of length $\frac{1}{3}$ (see figure). The process is repeated where I_{n+1} is obtained by replacing the middle third of each side of I_n by a new outward equilateral triangle with sides of length $\dfrac{1}{3^{n+1}}$. The limiting figure as $n \longrightarrow \infty$ is called the fractal snowflake.



a) Let L_n be the perimeter of I_n. Show that the $\lim_{n\to\infty} L_n = \infty$

b) Let A_n be the area of I_n. Find the $\lim_{n\to\infty} A_n$. It exists.

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8. (*6 points*) Zac places a_0 dollars in a savings account which earns 10% interest per month (its a good savings account!) That is during month zero Zac has a_0 dollars in the account and then in month one the bank places $\frac{a_0}{10}$ dollars into the account giving Zac a total of $a_0 + \frac{a_0}{10}$ dollars.

a) Write an recurrence relation for finding a_n the amount of dollars in the account during the n-th month.

b) Re-write the recurrence relation as an explicit function $f(x)$ which gives the amount in the account during month n.

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9. (*6 points*) Give the interval of convergence for the following power series.

a) $\sum_{k=0}^{\infty} k!x$

b) $\sum_{k=0}^{\infty} (-1)^k \frac{x^k}{5^k}$

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10. (*5 points*) Find the volume obtained when rotating the region R about the line $y = 2$. R is the region bounded inside $y = -x^2 + 1$ and $y = 4x^2$. You do not have to evaluate the integral.