NAME: ______________________________

Read all of the following information before starting the exam:

- **WRITE YOUR NAME AT THE TOP OF EACH PAGE (you will lose points otherwise)**
- **DO NOT WRITE ON THE FRONT OR BACK OF THE FIRST PAGE other that writing your name.**
- **Show all work and give explanations where needed. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).**
- **Use only the paper provided, your one page notes and a pen or pencil.**
- **Box your answers if possible.**
- **This test has 8 problems. and is worth 75 points, It is your responsibility to make sure that you have all of the pages!**
- **Good luck!**

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1. (*9 points*) Choose 3 of the following 4, make it clear which you choose, and where each problem begins and ends. Decide whether the following converge or diverge and give the test you used. If it converges and you give the value it converges to you get a bonus point.

a) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k\ln(k)}$

b) $\displaystyle\sum_{k=2}^{\infty} \frac{k^k}{k!}$

c) $\displaystyle\sum_{k=2}^{\infty} \frac{\arctan(k)}{k}$

d) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^{k+1}3^k}{4^{k+2}}$

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2. (*8 points*) Integrate 2 of the following 3, make it clear which you choose, and where each problem begins and ends.

a) $\int -3x^5\sqrt{1-x^3}\ dx$

b) $\int \sin^3(x)\cos^2(x)\ dx$

c) $\int e^x \sin(x)\ dx$

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3. (*8 points*) Choose 2 of the following 3, make it clear which you choose, and where each problem begins and ends. You do not have to show work for these.

a) Give the parametric equation for line segment starting at $(1, 2)$ and ending at $(5, -1)$, make sure to give the interval for t.

b) Give the parametric equation for the circle centered at $(1, 1)$ of radius 3 oriented clockwise, make sure to give the interval for t.

c) Graph the following parametric equation $x(t) = t^2 + 3t$ $y(t) = 1 + t$ for $-3 \leq t \leq 1$, and indicate with an arrow the orientation of the curve.

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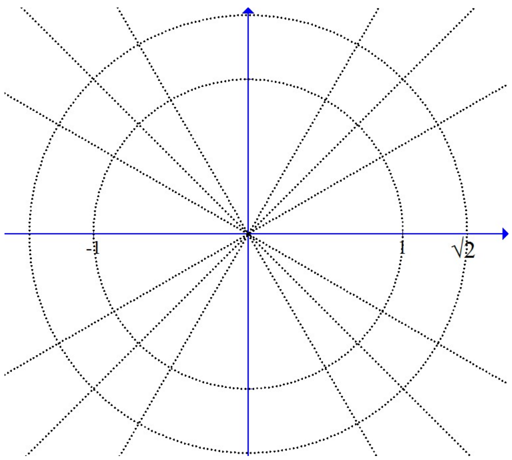
4. (*10 points*)

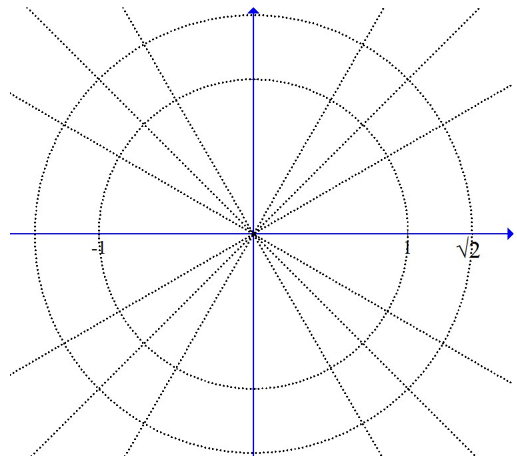
a) Graph the given equations.

b) On each graph label the point $\left(\frac{\pi}{4}, r\left(\frac{\pi}{4}\right)\right)$.

c) Give the area in the first quadrant bounded inside the curve $r(\theta) = \sqrt{2}\sin(3\theta)$ and outside the curve $r(\theta) = -\sqrt{2}\cos(3\theta)$.



$r(\theta) = \sqrt{2}\sin(3\theta)$



$r(\theta) = -\sqrt{2}\cos(3\theta)$

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5. (*12 points*) Let $f(x) = \frac{2}{3}x^{\frac{3}{2}}$ where $x \geq 0$.

a) Give the formula for the arc length of $f(x)$ from 0 to x and call this function $s(x)$. Hint: $s(x) = \int_0^x \sqrt{1 + (f'(t))^2}\, dt$.

b) Find the third degree Taylor polynomial for $s(x)$ centered at 0, and use the Taylor polynomial to estimate the arc length of $f(x)$ from 0 to 1. Hint: if you remember FTC1 your job is easier.

c) Use Taylor's remainder theorem to give an upper bound for $|R_3(1)|$.

d) Evaluate $s(1)$ directly. Explain why you would still need to approximate the value you got?

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6. (*12 points*)

a) For $n > 0$ an integer show that $\displaystyle\int y^n \ln(y)\ dy = \frac{y^{n+1}\ln(y)}{n+1} - \frac{y^{n+1}}{(n+1)^2} + c.$

b) Let R be the region bounded by $y = x^2$, $y = 1$, in the first quadrant. Compute the volume obtained when R is rotated about the y-axis.

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c) Compute $\lim_{r \to 0^+} r^2 \ln(r)$

d) Suppose the volume in part b) has a variable density, given by $\rho(y) = -\ln(y)$. So at the top of the shape the density is zero, and gets denser towards the bottom of the shape (much much denser.) Find the Mass of the shape. This is an improper integral and you must set it up correctly.

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7. (*8 points*)

a) Use the power series representation for $\frac{1}{1-x}$ to get a power series representation for $\ln(1+x)$, make sure to give the interval of convergence.

b) Use part a) to show that the harmonic series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}$ converges to $\ln(2)$. Do you think the harmonic series converges quickly or slowly to $\ln(2)$, explain your answer (this should be a short answer.)

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8. (*8 points*) Torricelli's law for fluid draining out of a cylindrical tank through a hole is given by $h'(t) = -2k\sqrt{h(t)}$ where $h(t)$ is the height of the fluid at time t for $t \geq 0$ and k is a constant which includes acceleration due to gravity (as well as a few other things.) Explain in physical terms what $h'(t)$ is and why it depends on $h(t)$. Let $k = \frac{1}{2}$ and solve for $h(t)$ with initial condition $h(0) = 25$ and graph $h(t)$ and give it's domain. Make sure your graph of $h(t)$ matches what you would expect (physically) if it does not, check the step where you used the initial condition.