NAME: ______________________________

Read all of the following information before starting the exam:

- **WRITE YOUR NAME AT THE TOP OF EACH PAGE (you will lose points otherwise)**
- **DO NOT WRITE ON THE FRONT OR BACK OF THE FIRST PAGE other that writing your name.**
- **Show all work and give explanations where needed. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).**
- **Use only the paper provided, your one page notes and a pen or pencil.**
- **Write you answer in the box provided.**
- **This test has 7 problems worth 70 points. In addition there is a bonus problem worth 5 points It is your responsibility to make sure that you have all of the pages!**
- **Good luck!**

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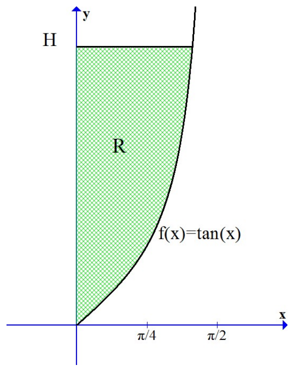
7	
Bonus	
total	

1. (*12 points*) Let R be the region bounded by $f(x) = \sqrt{x}$ and $g(x) = x^2$.

a) Find the area of the bounded region

b) Find the volume obtained when rotating R about the line $x = 2$

2. (*12 points*) The *Fancy Cone Company* makes fancy (not your normal) cones to order. The customer sends the *Fancy Cone Company* a fancy cone height H (in cone units) and the *Fancy Cone Company* makes them a fancy cone by rotating the region R (shown below) about the y-axis. The region R is bounded by $x = 0$ $y = H$ and $f(x) = \tan(x)$. Notice that these are not your normal cones...they are fancy!



a) You are hired by the *Fancy Cone Company* to build a function which takes H as an input and outputs the volume $V(H)$ of the cone (in cone units cubed). Give this function as an integral (set up but don't evaluate the resulting integral).

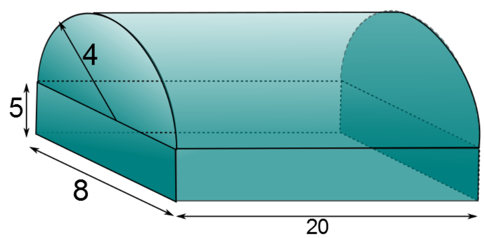
b) Compute $V'(1)$ and $V'(\frac{1}{\sqrt{3}})$.

c) Explain why $V'(1) > V'(\frac{1}{\sqrt{3}})$.

a)

b)

3. (*10 points*) Compute the work required to pump the water in the tank with shape given below to the top of the tank. All the dimension are listed and of the same units. Use ρ as the density of water and g as the gravity constant, assume the constants are in the correct units to match the units of length for the tank. Hint, in order to make the integrals manageable you should choose carefully where you place the "cross section" on the x, y-plane. Assume your pump hose always sits at the surface of the water.



4. (*12 points*) Compute the following integrals.

a) $\displaystyle\int_{-\frac{\pi}{4}}^{0} \frac{\sin^3(x)}{\cos(x)}\ dx$

b) $\displaystyle\int x^3\sqrt{x^2+1}\ dx$

c) $\displaystyle\int_0^{\pi} \sin(x)e^{x\sin x} + x\cos(x)e^{x\sin(x)}\ dx$

a)

b)

c)

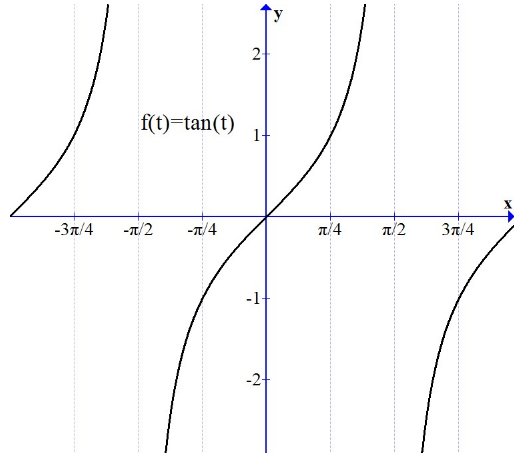
5. (*8 points*) Use a Riemann sum to compute the **area** between the x-axis and the function $f(x) = 4x^2 - 16$ from 0 to 2, use 4 rectangles and right endpoints. **Simplify your answer until you have an integer**. Is this an over or underestimate?

6. (*8 points*) State the fundamental theorem of calculus part 2, include the criteria a function must have to use FTC1. Explain why you **cant** integrate $f(x) = \frac{1}{x^2}$ on the interval -1 to 1.

7. (*5 points*) Given the function $g(x) = \int_{-\frac{\pi}{4}}^{\sqrt{x}} \tan(t)\, dt$

a) Give the linear approximation for $g(x)$ at $\frac{\pi^2}{16}$

b) Use the linear approximation from part a to estimate $g\left(\frac{\pi^2}{16} + \frac{1}{2}\right)$



a)

b)

8. (*5 points*) This is a bonus problem. It involves the shell method which we did not go over in class. If you know don't know the shell then only try this problem if you have extra time.

Find the volume obtained by rotating the shape bounded by $f(x) = \sin(x)$ and $y = 0$ about the y-axis.