NAME: _____

Read all of the following information before starting the exam:

- **WRITE YOUR NAME AT THE TOP OF EACH PAGE (you will lose points otherwise)**
- **DO NOT WRITE ON THE FRONT OR BACK OF THE FIRST PAGE other that writing your name.**
- **Show all work and give explanations where needed. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).**
- **Use only the paper provided, your one page notes and a pen or pencil.**
- **Write you answer in the box provided.**
- **This test has 8 problems worth 70 points. It is your responsibility to make sure that you have all of the pages!**
- **Good luck!**

1	
2	
3	
4	
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8	
total	

NAME: _____

1. (*12 points*) Let $f(x) = \frac{e^x - e^{-x}}{2}$

a) Find the fifth degree Taylor polynomial for $f(x)$ centered at zero, and use it to estimate $f(1)$. Hint $e^1 < 3$ and $\frac{1}{e} > 0$.

a)

b) Give an upper bound for the error of the estimate in part (a).

b)

c) Does $\lim_{n\to\infty} |R_n(1)| = 0$? What does this tell you about the relationship between $p_n(1)$ and $f(1)$.

d) For what values of $x \geq 0$ can $p_n(x)$ be made arbitrarily close to $f(x)$ (where n is chosen as large as needed). By this I mean for what values of x can $|R_n(x)|$ be made small? To answer this you should examine $\lim_{n\to\infty} |R_n(x)|$.

d)

2. (*8 points*) Evaluate the following:

1. $\int x^3 \sin(x^2)\, dx$

a)

2. $\int \frac{2}{x^2\sqrt{4-x^2}}\, dx$

b)

3. (*8 points*) Show that the series either converge or diverge. If they converge then give their value.

a) $\sum_{k=1}^{\infty} (-1)^{k+1} \left(\frac{3}{5}\right)^{2k}$

a)

b) $\sum_{k=1}^{\infty} \ln\left(\frac{k+1}{k}\right)$

b)

4. (*12 points*) **Choose 3** of the following, **state what convergence test you will use to determine if the series converges** and use the test to show the series converges or diverges. You do not have to say what the series converges to.

a) $\sum_{k=2}^{\infty} \frac{1}{k \ln(k)}$

a)

b) $\sum_{k=2}^{\infty} \sqrt{\frac{k}{k^3+1}}$

b)

c) $\sum_{k=2}^{\infty} \frac{1}{k^{\ln(k)}}$

c)

d) $\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$

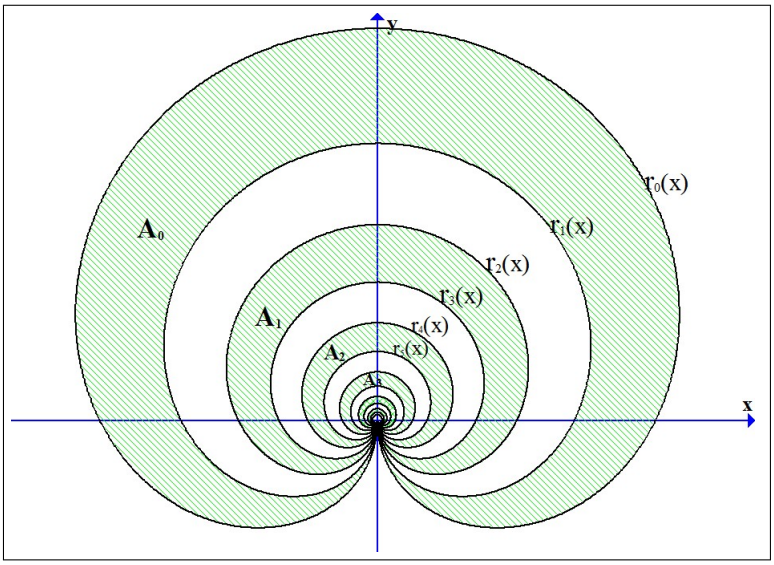
d)

e) $\sum_{k=1}^{\infty} \sin\left(\frac{1}{k}\right)$

e)

5. (*4 points*) Find the interval of convergence for $\sum_{k=1}^{\infty} \frac{x^k}{k}$

6. (*10 points*) Let $r_n(\theta) = \sqrt{\frac{2+2\sin(\theta)}{2^n}}$ for $n = 0, 1, 2, 3, ...$ be a polar graphs. See the picture below. Let A_n be the area between $r_{2n}(\theta)$ and $r_{2n+1}(\theta)$. For example A_0 is the area between $r_0(\theta)$ and $r_1(\theta)$ similarly A_1 is the area between $r_2(\theta)$ and $r_3(\theta)$.



a) Find A_n.

b) Find $\sum_{n=0}^{\infty} A_n$. Hint

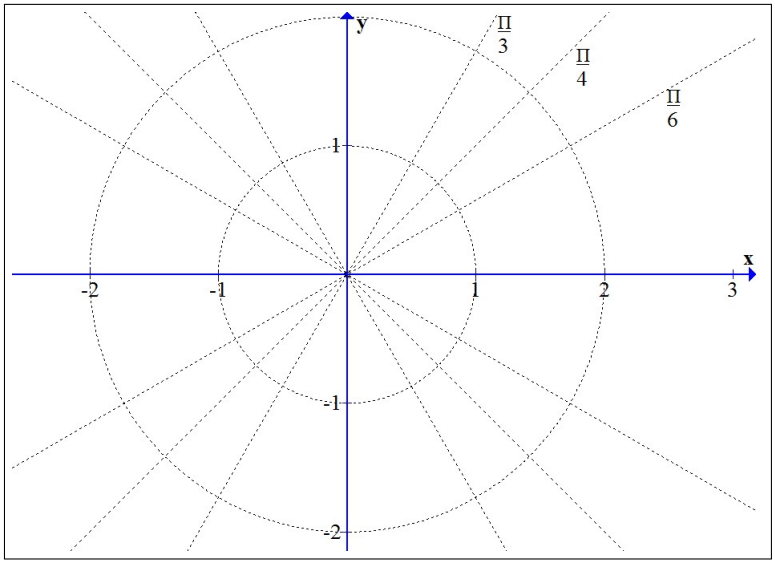
$$\left(\frac{1}{2^{2n}} - \frac{1}{2^{2n+1}}\right) = \frac{1}{2^{2n+1}}$$

.

a)

b)

7. (*8 points*) Graph the polar function $r(\theta) = 2\sin(3\theta)$ and on the graph provided. Then find the area inside bounded $r(\theta)$.



Area:

8. (*8 points*) Find the volume of the right square based pyramid with base length 10 units and height 20 units.