1. Integrate:

   (a) $\displaystyle\int_1^4 \frac{1}{x(x+3)}\ dx$

   (b) $\displaystyle\int_0^4 \frac{1}{\sqrt{x}}\ dx$

   (c) $\displaystyle\int_0^\infty \frac{1}{x\ln(x)}\ dx$

   (d) $\displaystyle\int e^{\frac{x}{2}}\cos(x)\ dx$

   (e) $\displaystyle\int \sin^3(2x)\cos^2(2x)\ dx$

   (f) $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x^3\sqrt{1-x^2}\ dx$

   (g) $\displaystyle\int x\ln(x)\ dx \qquad x>0$

   (h) $\displaystyle\int x^3\sin(x^2)\ dx$

   (i) $\displaystyle\int_0^\infty \frac{1}{(x+1)(x+2)}\ dx$

   (j) $\displaystyle\int \frac{2}{x^2\sqrt{4-x^2}}\ dx$

   (k) $\displaystyle\int_0^1 \frac{x^3+3x^2+2x+1}{x^2+3x+2}\ dx$

   (l) $\displaystyle\int_0^1 \tan^3(x)\sec^3(x)\ dx$

   (m) $\displaystyle\int_0^1 \ln(x)\ dx$

   (n) $\displaystyle\int \frac{x}{x^2+6x+10}\ dx$

   (o) $\displaystyle\int x^2\sqrt{4-x^2}\ dx$

   (p) $\displaystyle\int_0^1 \frac{x^4+x^3+x^2+3x+1}{x^3+x^2+x+1}\ dx$

   (q) $\displaystyle\int \frac{\sqrt{4x^2-2}}{x}\ dx$

   (r) $\displaystyle\int_0^1 \sin^3(\pi x)\cos^4(\pi x)\ dx$

   (s) $\displaystyle\int_0^1 \arctan(x)\ dx$

2. Let $f(x)=x^2-1$ find:

   (a) the area under the curve from -2 to 2 of $|f(x)|$.

   (b) the arc length from -2 to 2 of $f(x)$, you can use the fact that $\displaystyle\int_{\arctan(-4)}^{\arctan(4)} \sec(x)\ dx = \ln\left(\frac{\sqrt{17}+4}{\sqrt{17}-4}\right)$

(c) What is the arc length of $|f(x)|$ from -2 to 2?(You should not spend more than 30 seconds on this problem.) Explain your answer.

3. Give the parametric equations for the following curves.

   (a) The straight line segment between $(-3, 100)$ and $(1, 1)$ where time zero corresponds to $(-3, 100)$ and time one corresponds to $(1, 1)$.

   (b) The circle centered at $(1, 1)$ oriented in the counterclockwise direction which completes one revolution in 1 unit of time.

4. Let $r(\theta) = 1$ and $r(\theta) = 1 + \cos(\theta)$ be functions in polar coordinates. Find the area bounded between the two curves.

5. Do the following:

   (a) Write the polar equation for the circle centered at $(1, 0)$ of radius 1 and then use polar integration to compute the area of the circle. Does this match what you expect the area of a circle of radius one to be.

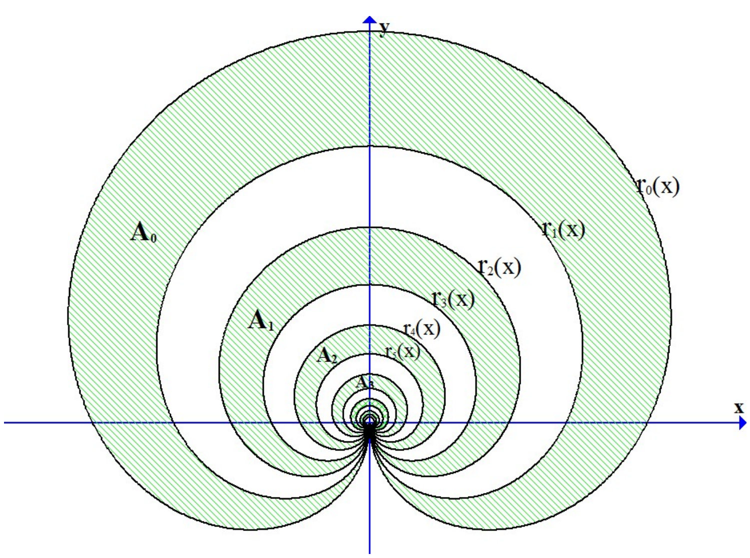
   (b) A circular slab of cement of unit radius 1 is surrounded by grass and a goat is tied with a rope of length a where $0 \le a \le 2$ to the edge of the slab. What is the area of the grassy region the goat can graze from?

   (c) Does your answer make sense if $a = 0$ or $a = 2$?

   (d) If $a \ge 2$ can you find the grazing area?

6. Graph the polar function $r(\theta) = 2\sin(3\theta)$ and find the area inside bounded $r(\theta)$.

7. Let $r_n(\theta) = \sqrt{\frac{2+2\sin(\theta)}{2^n}}$ for $n = 0, 1, 2, 3, ...$ be a polar graphs. See the picture below. Let A_n be the area between $r_{2n}(\theta)$ and $r_{2n+1}(\theta)$. For example A_0 is the area between $r_0(\theta)$ and $r_1(\theta)$ similarly A_1 is the area between $r_2(\theta)$ and $r_3(\theta)$.



a) Find A_n.

b) Find $\sum_{n=0}^{\infty} A_n$.

8. Graph the following
   (a) $x(t) = \cos(t)$, $y(t) = \sin(t)$ for $0 \le t \le \frac{3\pi}{2}$
   (b) $x(t) = \sin(t)$, $y(t) = \cos(t)$ for $0 \le t \le \pi$
   (c) $x(t) = \cos(t) - 1, \quad y(t) = \sin(t) - 1$
   (d) $x(t) = t\cos(t)$, $y(t) = t\sin(t)$ for $0 \le t \le 5\pi$
   (e) $x(t) = t\cos(t)$, $y(t) = t\sin(t)$ for $-5\pi \le t \le 0$
   (f) $x(t) = t^2, \quad y(t) = t$
   (g) $x(t) = \sin(t), \quad y(t) = t$
   (h) $x(t) = t^2 + 2, \quad y(t) = t^2$

9. Do the following:
   a) Show that $y = e^{2x}$ is a solution to the differential equation $y'(x) = yx$.
   b) Solve the initial value problem for the differential equation $y'(x) = x(y-1)$ with $y(0) = 3$.
   c) Solve the initial value problem for the differential equation $h'(t) = -2\sqrt{h(t)}$ with $h(0) = 36$.
   d) Which of the following is $f(x) = \displaystyle\int_a^x te^t \, dt$ a solution to?
      i. $f''(x) - f'(x) = e^x$
      ii. $2f'(x) = f(x) - 1$
      iii. $f''(x) + f'(x) = x$

10. Graph the equations $r(\theta) = 2 + 2\cos(\theta)$ and $r(\theta) = 2 + 2\sin(\theta)$. On each graph label the point $\left(\frac{\pi}{4}, r\left(\frac{\pi}{4}\right)\right)$. Give the area in the first quadrant bounded inside the curve $r(\theta) = 2 + 2\cos(\theta)$ and outside the curve $r(\theta) = 2 + 2\sin(\theta)$.

11. Let $f(x) = \frac{2}{3}x^{\frac{3}{2}}$ where $x \ge 0$.
    a) Give the formula for the arc length of $f(x)$ from 0 to x and call this function $s(x)$.
    b) Find the third degree Taylor polynomial for $s(x)$ centered at 0, and use the Taylor polynomial to estimate the arc length of $f(x)$ from 0 to 1. Hint: if you remember FTC1 your job is easier.
    c) Use Taylor's remainder theorem to give an upper bound for $|R_3(1)|$.
    d) Evaluate $s(1)$ directly. Explain why you would still need to approximate the value you got.

12. Do the following:
    a) For $n > 0$ an integer show that $\displaystyle\int y^n \ln(y) \, dy = \frac{y^{n+1}\ln(y)}{n+1} - \frac{y^{n+1}}{(n+1)^2} + c.$
    b) Let R be the region bounded by $y = x^2$, $y = 1$, in the first quadrant. Compute the volume obtained when R is rotated about the y-axis.
    c) Compute $\displaystyle\lim_{r \to 0^+} r^2 \ln(r)$
    d) Suppose the volume in part b) has a variable density, given by $\rho(y) = -\ln(y)$. So at the top of the shape the density is zero, and gets denser towards the bottom of the shape (much much denser.) Find the **mass** of the shape. This is an improper integral and you must set it up correctly.

13. Let R be the region bounded by $f(x) = \cos(x^2)$, $x = 0$, $x = \sqrt{\frac{\pi}{2}}$ and the x-axis. Compute the mass of the shape obtained by rotating R about the x-axis if the shape has a density function $\rho(x) = x^3$.

14. Graph and find the area bounded inside $r(\theta) = 2\cos(2\theta)$ but outside $r(\theta) = 2\sin(2\theta)$ in the first quadrant.

15. Give a parametric equation for a circle of radius 2 oriented in the clockwise direction centered at $(0, -2)$ with initial point $(0, 0)$.

16. Graph the following functions. Label the x and y axis values or points on the curve (make it clear you know what the graph looks like.)

    (a) Parametric equation $x(t) = 1 + t$ $y(t) = 1 - 2t$ for $-1 \leq t \leq \frac{1}{2}$

    (b) Parametric equation $x(t) = \tan(t)$ $y(t) = \sec^2(t) - 1$ for $\frac{\pi}{4} \leq t < \frac{\pi}{2}$. Hint: eliminate the parameter.

    (c) The polar function $r(\theta) = \dfrac{2}{2\cos(\theta) + 3\sin(\theta)}$. Hint: convert to a Cartesian function.

17. Evaluate $\displaystyle\int_0^1 x\ln(x)\, dx$. Note this is an improper integral. You must show all work (show the computations involved in computing the integral and limits.)

18. Compute $\displaystyle\sum_{k=2}^{1000} \frac{1}{k^2 + 7k + 12}$. Then compute the infinite series $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k^2 + 7k + 12}$.

19. Compute the arc length of $f(x) = \dfrac{e^x + e^{-x}}{2}$ from 0 to $\ln(2)$.

20. **Use a series convergence test** to determine if the following series converge. You must state which test you use and then apply it correctly. A bonus point if you state what the series converges to.

    a) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k\ln(k)}$

    b) $\displaystyle\sum_{k=1}^{\infty} \sqrt{\frac{k}{k^3 + 1}}$

    c) $\displaystyle\sum_{k=3}^{\infty} \frac{(-1)^k 2^{k+1}}{3^{2k}}$