1. Give the interval of convergence for the following power series.

   a) $\sum_{k=0}^{\infty} k!x^k$

   b) $\sum_{k=0}^{\infty} (-1)^k \frac{x^k}{5^k}$

   c) $\sum_{k=1}^{\infty} \frac{x^k}{k}$

   d) $\sum_{k=1}^{\infty} \frac{(x-1)^k}{k!}$

   e) $\sum_{k=0}^{\infty} \frac{(-1)^k (x-2)^k}{4^k}$

2. Find a power series representations for the following (centered at zero) and state their radius of convergence.

   (a) $f(x) = \frac{1}{1+x^2}$

   (b) $f(x) = \ln(1-x)$

   (c) $f(x) = \arctan(x)$

   (d) $f(x) = e^{x^2}$

   (e) $f(x) = \frac{1}{(1-x)^2}$

3. Use a Riemann sum to compute the **area** between the x-axis and the function $f(x) = 4x^2 - 16$ from 0 to 2, use 4 rectangles and right endpoints. **Simplify your answer until you have an integer**. Is this an over or underestimate?

4. Use a Riemann sum with 4 rectangles and bottom end points to compute the volume of the shape obtained when rotating the region R about the y-axis. R is the region in the first quadrant bound by $y^4 + x^2 = 1$. Is this an over estimate or an under estimate?

5. Give the linear approximation for the function $A(x) = \int_0^x e^{\frac{-t^2}{2}}\, dt$ centered at zero. Does it make sense that your linear approximation has a positive slope, explain?

6. Use the fundamental theorem of calculus part 1, the chain rule and the properties of integration to derive the formulas:

   (a) $\frac{d}{dx}\int_a^{g(x)} f(t)\, dt = f(g(x))g'(x)$, $f(t)$ is continuous and $g(x)$ is differentiable.

   (b) $\frac{d}{dx}\int_{h(x)}^{g(x)} f(t)\, dt = f(g(x))g'(x) - f(h(x))h'(x)$, $f(t)$ is continuous, $g(x)$ and $h(x)$ are differentiable.

7. Give the derivative of $f(x) = e^x \sin(x)$, and evaluate $\int_0^{\pi} e^x(\sin(x) + \cos(x))\, dx$

8. Give the following anti-derivatives/evaluate.

   (a) $\int x \sin(x^2)\, dx$

   (b) $\int x^3 \sqrt{x^2+1}\, dx$

   (c) $\int_0^1 \frac{x}{x-2}$

(d) $\int_0^1 x(x^2+3)^{100}\,dx$

(e) $\int x(x^2+2x+1)^2\,dx$

(f) $\int_{-\pi}^{\pi} x^3\cos(x)\,dx$

9. Give the volumes of the shape obtained by rotating the region R about the given axis.

   (a) R is the region bounded by $f(x)=\sin(x)$, $g(x)=x$ and $x=\pi$, about the x axis.

   (b) R is the region bounded by $f(x)=\sin(x)$, $g(x)=x$ and $x=\frac{\pi}{2}$, about the $y=\frac{\pi}{2}$ axis.

   (c) R is the region bounded by $f(x)=\sin(x^2)$, $g(x)=x$ and $x=\sqrt{\pi}$, about the y axis.

   (d) R is the region bounded by $f(y)=\sec(y)$, $x=0$, $y=\frac{-\pi}{4}$ and $y=\frac{\pi}{4}$ about the y axis.

10. Find the volume of the right square based pyramid with height 10 and base 8.

11. Find the volume of the shape with circular base of radius r with equilateral triangle cross sections that are perpendicular to the base.

12. Derive the volume of a sphere with radius R.

13. Find the work required to lift the liquid with density constant ρ out of a right square based pyramid with height H and base B to a height k above the tank. Use g as the gravity constant.

14. Find the work required to lift the liquid with density constant ρ out of a an inverted cone with base radius R and height H to the top of the tank. Use g as the gravity constant.

15. Find the work required to lift the liquid with density constant ρ out of a trough with triangular cross section and length L to a height k above the tank. The triangular cross section is an equilateral with side length S. Use g as the gravity constant.

16. Find the Mass of the shape obtained by rotating the region R about the y axis when the shape has density function $\rho(x,y,z)=\rho(y)=y^2$. The region R is the area bounded by $f(x)=x^2$ and $g(x)=\sqrt{x}$.

17. Let R be the region bounded by $f(x)=\sin(x^2)$, $y=0$, $x=0$ and $x=\sqrt{\pi}$. Find the mass of the shape obtained when R is rotated about the x axis if the shape has density function $\rho(x,y,z)=\rho(x)=x$.

18. Let R be the region bounded by $f(x)=\sqrt{(x-1)\sin(x-1)}$, $x=1$, $x=\pi+1$ and the x-axis. Set up the integral to find the volume of the solid generated by rotating R about the x-axis (you do not need to do the integral.)

19. Let the sequence $\{a_n\}_0^\infty$ be the sequence defined by a_n is the arc length of the curve $f(x)=x^n$ on $[0,1]$. Compute a_0 and a_1 and write a formula for a_n involving an integral. Make an argument (a picture with a few sentences will suffice) that the sequence is positive, increasing and bounded. Show the sequence a_n converges, state any theorems you use. What do you think the sequence converges to?

20. Find the total force on a dam due to water pressure. The dam is a symmetric trapezoid with height H, length T on its top edge, and length B along its bottom edge. Use ρ as the density of water and g as the gravitational constant.

21. Find the total force on a dam due to water pressure. The lower edge of the dam is defined by the parabola $\frac{x^2}{16}$ with the bottom of the parabola at $y=0$ and the top edge at position $y=25$ is a straight line. Use ρ as the density of water and g as the gravitational constant.