You are responsible for all material covered in class, on the homework, and on all quizzes. One question on the midterm may come from a written quiz, so study those. Below is a study guide.

Taylor/Power Series:

- Know what a Taylor and Maclaurin series are.
- Taylor Series you should know, with the radii of convergence:
  - e^x centered at 0
  - $\log(1-x)$ centered at 0
  - $\sin(x)$ and $\cos(x)$ centered at 0
  - $\tan(x)$ centered at 0
  - $\log(x)$ centered at 1
  - $\frac{1}{1-x}$ centered at 0
- Know how to manipulate power series representations of functions to obtain new power series representations of functions with the interval of convergence. For example, find power series representations for:
  - $f(x) = g(3x^3)$ where g is a function listed above.
  - $f(x) = \ln(1+x^2)$ centered at zero.
  - $f(x) = \arctan(x)$ centered at zero.
- Know how to find functions represented by a given power series. Specific examples:
  - Find the function represented by $\sum_{k=1}^{\infty} kx^k$ by differentiating a power series you know. What is the interval on which the power series converges to the function?
  - Show $\int e^x dx = e^x + C$ using power series.

Riemann Sums:

- Definition of the Riemann sum.
- How to compute Riemann sums using left and right endpoints.
- Example: Compute Riemann sum using left endpoints of $f(x) = x^2$ from 1 to 3 and $n = 3$.
- How increasing functions v.s. decreasing effect whether left/right endpoints give an over or underestimate.
- See Chapter 5 on Riemann Sums using Sigma Notation for formulas for computing sums.

Fundamental Theorem of Calculus:

- Know the definition of the Area Function.
- Know the definition and implications of the Fundamental Theorem Part I:

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

- Know the definition and implications of the Fundamental Theorem Part II:

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Applications of Riemann Sums/Integrals:

- Given a continuous velocity function, compute distance traveled over a given time frame. Same for acceleration, population growth rates, and other application problems from Section 6.1.
- Given discrete data points for velocity, approximate the distance traveled (same for other applications).
- Area between curves
- u-substitution.

Computing Volumes:

- Understand how to find the volume of various shapes using cross-sectional areas.
- Examples:
  - Volume of a square cross sections on circle base
  - Volume of pyramid
  - Volume of equilateral triangles on circle base (gnome/elf hat)
- Volumes by rotation. Using integration with respect to x and y.
- Disk, Washer, and Shell Methods.
- Examples:
  - The region bounded by $f(x) = \sqrt{x}$, $y = 0$ and $x = 3$ rotated about x-axis.
  - The region bounded by $f(x) = \sqrt{x}$, $y = -1$ and $x = 3$ rotated about $y = -1$.
  - The region bounded by $f(x) = \sqrt{x}$, $y = 1$ and $x = 3$ rotated about $y = 1$.
  - The region bounded by $f(x) = \sqrt{x}$, $y = 0$ and $x = 3$ rotated about $y = -1$.
  - The region bounded by $f(x) = \sqrt{x}$, $y = -1$ and $x = 3$ rotated about $y = -1$.
  - The region bounded by $y = x^2$, $y = 9x^2$ and $y = 4$ rotated about $x = 6$.
  - The region bounded by $f(x) = \sqrt{x}$, $y = 0$, and $x = 3$ rotated about $y = 5$.
  - The region bounded by $y = |x|$, $x = -(y+1)^2$, $y = -2$ and $y = 3$ rotated about $x = 4$.

Mass/Work/Force:

- Know how to compute the mass of objects with a density function in one variable.
- Know how to compute the work required to lift water out of a tank (in terms of density, volume, gravity and distance). There are many examples in the book.
- Know how to compute the work required to stretch or compress a spring with given spring constant.
- Know how to compute the force caused by hydrostatic pressure (see the dam examples).

Arc length:

- Know how to apply the arc length formula.
- Examples: Find the arc length of the following functions on the interval given

  a) $f(x) = (x+1)^{\frac{3}{2}}$ from -1 to 1

  b) $f(x) = \dfrac{e^x + e^{-x}}{2}$ from 0 to $\ln(2)$

  c) $f(x) = \sqrt{1-x^2}$ from -1 to 1