DISCLAIMER: The following problems are just examples of types of problems that **MAY** appear on the exam. They will not cover every possible topic and not all topics discussed here may show up on the exam. Also, there may be typos and errors, so if something does not seem right, please ask.

1. State whether the following infinite series converge or diverge, there is no need to compute their limits.

   a) $\sum_{k=0}^{\infty} \sin\left(\frac{k}{k+1}\right)$

   b) $\sum_{k=1}^{\infty} \arctan(k)$

   c) $\sum_{k=0}^{\infty} \frac{k^k}{2^{k!}}$

   d) $\sum_{k=1}^{\infty} \frac{1}{k\ln^2(k)}$

2. State whether the following infinite series converge or diverge, **it they converge determine their limit**.

   a) $\sum_{k=3}^{\infty} (-1)^k \; \frac{2^{(2k+1)}}{16^{(k+1)}}$

   b) $\sum_{k=1}^{\infty} \ln\left(\frac{k}{k+1}\right)$

3. Let $f(x) = e^{-\frac{x}{2}}$

   a) Give the fourth degree Taylor polynomial for $f(x)$ centered at zero.

   b) Use the Taylor polynomial from part a) to estimate e.

   c) Give an upper bound for the error of your estimate (keep your estimates within reason.)

4. The fractal snowflake is constructed as follows: Let I_0 be an equilateral triangle with sides of length l. The figure I_1 is obtained by replacing the middle third of each side of I_0 by a new outward equilateral triangle with sides of length $\frac{1}{3}$ (see figure). The process is repeated where I_{n+1} is obtained by replacing the middle third of each side of I_n by a new outward equilateral triangle with sides of length $\frac{1}{3^{n+1}}$. The limiting figure as $n \longrightarrow \infty$ is called the fractal snowflake.

   a) Let L_n be the perimeter of I_n. Show that the $\lim_{n\to\infty} L_n = \infty$

   b) Let A_n be the area of I_n. Find the $\lim_{n\to\infty} A_n$. It exists.

5. Zac places a_0 dollars in a savings account which earns 10% interest per month (its a good savings account!) That is during month zero Zac has a_0 dollars in the account and then in month one the bank places $\frac{a_0}{10}$ dollars into the account giving Zac a total of $a_0 + \frac{a_0}{10}$ dollars.

   a) Write an recurrence relation for finding a_n the amount of dollars in the account during the n-th month.

   b) Re-write the recurrence relation as an explicit function $f(n)$ (here n must be a positive integer) which gives the amount in the account during month n.

6. Let $f(x) = \frac{e^x - e^{-x}}{2}$

   a) Find the fifth degree Taylor polynomial for $f(x)$ centered at zero, and use it to estimate $f(1)$. Hint $e^1 < 3$ and $\frac{1}{e} > 0$.

   b) Give an upper bound for the error of the estimate in part (a).

c) Does $\lim_{n\to\infty} |R_n(1)| = 0$? What does this tell you about the relationship between $p_n(1)$ and $f(1)$.

d) For what values of $x \geq 0$ can $p_n(x)$ be made arbitrarily close to $f(x)$ (where n is chosen as large as needed). By this I mean for what values of x can $|R_n(x)|$ be made small? To answer this you should examine $\lim_{n\to\infty} |R_n(x)|$.

7. Show that the series either converge or diverge. If they converge then give their value.

a) $\sum_{k=1}^{\infty} (-1)^{k+1} \left(\frac{3}{5}\right)^{2k}$

b) $\sum_{k=1}^{\infty} \ln\left(\frac{k+1}{k}\right)$

8. **Choose 3** of the following, **state what convergence test you will use to determine if the series converges** and use the test to show the series converges or diverges. You do not have to say what the series converges to.

a) $\sum_{k=2}^{\infty} \frac{1}{k\ln(k)}$

b) $\sum_{k=2}^{\infty} \sqrt{\frac{k}{k^3+1}}$

c) $\sum_{k=2}^{\infty} \frac{1}{k^{\ln(k)}}$

d) $\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$

e) $\sum_{k=1}^{\infty} \sin\left(\frac{1}{k}\right)$

9. Choose 5 of the following to do and make it clear which you have chosen. Decide whether the following series converge. If they converge and you can decide what they converge to then give the value. You are expected to know for which convergent series you can find the limit.

(a) $$\sum_{k=4}^{\infty} \frac{(-1)^{k+1}}{3^k}$$

(b) $$\sum_{k=1}^{\infty} \frac{\arctan(k)}{k^2}$$

(c) $$\sum_{k=0}^{\infty} (-1)^k \left(\frac{k^2+2k+2}{k^3+15k+1}\right)$$

(d) $$\sum_{k=1}^{1003} \frac{1}{k} - \frac{1}{k+2}$$

(e) $$\sum_{k=0}^{\infty} \frac{k^k}{(k+1)!}$$

(f) $$\sum_{k=0}^{\infty} \frac{k}{e^{k^2}}$$

10. Find the 3-rd order Taylor polynomial for $f(x) = x\ln(x)$ with center 1.