

You are responsible for all material covered in class, on the homework, and on all quizzes. One question on the midterm may come from a written quiz, so study those. Below is a study guide.

## L'Hopital's Rule

- Know how to apply L'Hopital's rule in normal and in indeterminate forms.
- Examples: Be able to do the following problems:

1.  $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$ .

2.  $\lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right)$ .

3.  $\lim_{x \rightarrow \infty} (x - \sqrt{x^2 - 3x})$ .

4.  $\lim_{x \rightarrow 0} x^x$ .

5.  $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^x$  for a constant  $a$ .

## Sequences

- Know the definition of a sequence.
- Know what it means for a sequence to converge.
- Know how to use functions to show the convergence of a sequence (p. 607).
- Know the properties of limits (p. 607).
- Know the definitions for increasing, decreasing, strictly increasing, strictly decreasing, monotonic and bounded sequences.
- Know what a geometric sequence is and when it converges/diverges.
- Know the statement of the squeeze theorem.
- Know that bounded monotonic sequences converge.
- Review problems from Section 8.2.

## Series

- Know the definition of an infinite series.
- Know what the sequence of partial sums for a series is.
- Know what it means for an infinite series to converge.
- Know what a geometric series is and when it converges/diverges.

- Know how to use the geometric series to change a repeating decimal into a fraction (see Example 2 on p. 622).
- Know what a p-series is and for what value of  $p$  it converges.
- Know what an alternating series is and under what conditions it converges.
- Know how to handle telescoping series, for example:
  1.  $\sum_{k=1}^{\infty} \left(\frac{1}{3^k} - \frac{1}{3^{k+1}}\right)$ ,
  2.  $\sum_{k=1}^{\infty} \ln\left(\frac{k+1}{k+3}\right)$ .
- Know properties of convergent series (p. 636).
- Know the statement of and how to apply the divergence test.
- Know the statement of and how to apply the ratio test.
- Know the statement of and how to apply the root test.
- Know the statement of and how to apply the comparison and limit comparison test.
- We won't give a list of problems to practice, Chapter 8 is full of excellent examples (try the odd version of the assigned homework problems).

## Linear Approximation

- Know how to find the linear approximation of a function at a point.
- Know under what conditions a linear approximation is a "good" approximation for a function.
- For example, use linear approximations to estimate the values of:
  1.  $\sqrt{1.003}$
  2.  $\tan^{-1}(1.001)$

## Newton's Method

- Know what the sequence of Newton iterations is for a given function (See Section 4.8, particularly Figure 4.86).

- Know how the method is derived using linear approximations.
- Be able to explain with the use of a graph, whether using a given starting guess will lead to a convergent or divergent sequence.
- Know how to find bounds for the error terms of Taylor polynomials (p. 668-669).
- Look over problems in Section 9.1.
- Examples: Give the Taylor polynomials for the following.

### Taylor polynomials/series

- Know the definition of and how to compute an  $n$ -th degree Taylor polynomial (p. 664).
- Look over example 2 on p. 664.
- 1.  $f(x) = e^x$  centered at 0 of degree  $n$ .
- 2.  $f(x) = \ln(x)$  centered at 1 of degree  $n$ .
- 3.  $f(x) = \sin(x)$  centered at  $\pi$  of degree 5 and of degree 6.