Math 34 Reality Check Fall 2013

Name KEY Section _____

1. Simplify $\frac{a_{n+1}}{a_n}$ for the following a_n's.

(a) $a_n = \frac{(2n)!x^n}{3^n}$

$$\frac{(2(n+1))!\,x^{n+1}}{3^{n+1}} \cdot \frac{3^n}{(2n)!\,x^n} = \frac{x^{n+1}}{x^n} \cdot \frac{3^n}{3^{n+1}} \cdot \frac{(2n+2)!}{(2n)!}$$

$$= \frac{x}{3} \cdot \frac{(2n+2)(2n+1)(2n)!}{(2n)!} = \frac{x}{3}(2n+2)(2n+1)$$

(b) $a_n = \frac{4^{2n}x^n}{(n+1)(n!)^2}$

$$\frac{4^{2(n+1)}\,x^{n+1}}{(n+1+1)((n+1)!)^2} \cdot \frac{(n+1)(n!)^2}{4^{2n}\cdot x^n} = \frac{x^{n+1}}{x^n} \cdot \frac{4^{2n+2}}{4^{2n}} \cdot \frac{(n+1)}{(n+2)} \cdot \frac{(n!)(n!)}{(n+1)!\,(n+1)!}$$

$$= x \cdot 4^2 \cdot \frac{n+1}{n+2} \cdot \frac{n!\,n!}{(n+1)n!\,(n+1)n!} = x \cdot 4^2 \cdot \frac{n+1}{n+2} \cdot \frac{1}{(n+1)(n+1)} = \frac{x \cdot 16}{(n+2)(n+1)}$$

(c) $a_n = \frac{5^n x^{2n}}{3n!}$

$$\frac{5^{n+1}\cdot x^{2(n+1)}}{3\,(n+1)!} \cdot \frac{3n!}{5^n \cdot x^{2n}} = \frac{x^{2n+2}}{x^{2n}} \cdot \frac{5^{n+1}}{5^n} \cdot \frac{3n!}{3(n+1)n!} = \frac{x^2 \cdot 5}{n+1}$$

2. Solve the following for x.

(a) $|2x+3| < 1$

$$-1 < 2x+3 < 1$$
$$-4 < 2x < -2$$
$$-2 < x < -1$$

(b) $|\frac{x^2}{9}| < 1$

$$\frac{|x^2|}{9} < 1$$
$$|x^2| < 9$$
$$|x|^2 < 9$$
$$|x| < 3 \quad \text{or} \quad -3 < x < 3$$

(c) $|\frac{\sqrt{x}}{4} - 1| < 1$

$$-1 < \frac{\sqrt{x}}{4} - 1 < 1$$
$$0 < \frac{\sqrt{x}}{4} < 2$$
$$0 < \sqrt{x} < 8$$
$$0 < x < 64$$

(d) $|27x^3| < 1$

$$27|x^3| < 1 \qquad |x^3| < \frac{1}{27} \qquad |x|^3 < \frac{1}{27} \qquad |x| < \sqrt[3]{\frac{1}{27}}$$
$$|x| < \frac{1}{3}$$
$$-\frac{1}{3} < x < \frac{1}{3}$$