Math 34 Quiz 7a Fall 2013

Name KEY Section ______

Determine whether the following series is absolutely convergent, conditionally convergent, or divergent. Justify your answer. State and check hypotheses of any test, rules or theorem you use.

$$\sum_{n=1}^{\infty}(-1)^n\frac{n}{400+n^2}$$

$\boxed{1}$ AbC? $\displaystyle\sum_{n\geq1}^{\infty}\frac{n}{400+n^2} \sim \sum_{n\geq1}^{\infty}\frac{n}{n^2} = \sum_{n\geq1}^{\infty}\frac{1}{n}$

$\underline{\text{LCT}}$ Let $\displaystyle L = \lim_{n\to\infty}\frac{n}{400+n^2}\cdot\frac{n}{1} = \lim_{n\to\infty}\frac{n^2}{400+n^2} = \lim_{n\to\infty}\frac{1}{400/n^2+1} = 1$

Since $0 < 1 < +\infty$ and since $\sum\frac{1}{n}$ is the divergent harmonic series (p-series, $p=1\leq1$) our series is <u>not</u> AbC.

$\boxed{2}$ CC? $\displaystyle\sum_{n\geq1}^{\infty}(-1)^n\frac{n}{400+n^2}$

A.S.T ① $\displaystyle\lim_{n\to\infty}\frac{n}{400+n^2} = \lim_{n\to\infty}\frac{\overset{\to0}{(1/n)}}{\underset{\to0}{(400/n^2)}+1} = 0$ ✓

② $\dfrac{n}{400+n^2} > 0$ for $n\geq1$ ✓

③ Show $\left\{\dfrac{n}{400+n^2}\right\}$ is eventually decreasing;

Let $f(x) = \dfrac{x}{400+x^2}$, $x\geq1$

$$f'(x) = \frac{(400+x^2)\cdot1 - x(2x)}{(400+x^2)^2} = \frac{400-x^2}{(400+x^2)^2} < 0 \text{ for } x>20 ✓$$

∴ $f(x)$ is decreasing for $x>20$ so $\left\{\frac{n}{400+n^2}\right\}$ decreases for $n>20$

So our series is CONDITIONALLY CONVERGENT