Math 34 Quiz 6a Fall 2013

Name KEY Section ______

Determine whether the following series converges or diverges. Justify your answer. State and check hypotheses of any test, rules or theorem you use.

$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ Integral Test Let $f(x) = \frac{1}{x(\ln x)^2}$, $x \geq 2$

Then (1) f is continuous and positive, $x \geq 2$.

(2) Need to show f is eventually decreasing.

option 1 $f(x) = \left[x(\ln x)^2\right]^{-1}$

$$f'(x) = -\left[x(\ln x)^2\right]^{-2}\left[1\cdot(\ln x)^2 + \cancel{x}\cdot 2(\ln x)\cdot\cancel{\tfrac{1}{x}}\right] < 0$$

$+ \quad + \quad +$ for $x \geq 2$

Since $f'(x) < 0$ for $x > 2$, f is decreasing for $x \geq 2$

option 2 Since x, $(\ln x)^2$ are both positive and increasing for $x \geq 2$, $x(\ln x)^2$ is increasing for $x \geq 2$ so $\frac{1}{x(\ln x)^2}$ is decreasing for $x \geq 2$.

So we can use the integral test!

$$\int_2^{\infty} \frac{dx}{x(\ln x)^2} = \lim_{t\to\infty} \int_2^t \frac{dx}{x(\ln x)^2} = \lim_{t\to\infty}\left[-\frac{1}{(\ln x)}\right]_2^t = \lim_{t\to\infty}\left[-\frac{1}{(\ln t)} + \frac{1}{\ln 2}\right] = \frac{1}{\ln 2}$$

$-\frac{1}{(\ln t)} \to 0$

$\left[\text{"form" } \int \frac{du}{u^2},\ \int \frac{du}{u^2} = -\frac{1}{u} + C\right]$

Since the integral converges, the series converges by the integral test.