**Math 34** | **TUFTS UNIVERSITY** | **May 2, 2014**
**Calculus II** | **Department of Mathematics** | **All sections**

**Final Exam**

No books, notes, **or calculators**. **TURN OFF YOUR CELL PHONE. ANYONE CAUGHT WITH THEIR CELL PHONE ON WILL BE GIVEN A 10 POINT DEDUCTION.** Cross out what you do not want us to grade. You **must** show work to receive full credit. Please try to write neatly. You need not simplify your answers unless asked to do so. You should evaluate standard trigonometric functions like $\tan(\pi/3)$. You are not allowed to quote results about growth rates. You are required to **sign** your exam book. With your signature, you pledge that you have neither given nor received assistance on this exam.

---

| Problem | Point Value | Points |
|---|---|---|
| 1 | 12 | |
| 2 | 8 | |
| 3 | 12 | |
| 4 | 7 | |
| 5 | 8 | |
| 6 | 8 | |
| 7 | 6 | |
| 8 | 8 | |
| 9 | 12 | |
| 10 | 5 | |
| 11 | 6 | |
| 12 | 8 | |
| | 100 | |

1. (12 points) Integrate.

(a) $\displaystyle\int \cos^3(x)\,dx$

(b) $\displaystyle\int \frac{dx}{x^2 - 3x - 4}$

2. (8 points) Find the limits of each of the following sequences or state that they do not exist.

(a) $a_n = \dfrac{5 \sin n}{n^3}$

(b) $a_n = n \ln \left(1 + \dfrac{4}{n}\right)$

3. (12 points) Determine whether each of the following series converges or diverges. Justify your answer. State and check hypotheses of any test, rules or theorems you use. You may *not* simply quote a theorem.

(a) $$\sum_{k=1}^{\infty} \frac{24k^2 + 30k}{k^3 + 1}$$

(b) $$\sum_{k=1}^{\infty} \frac{(-1)^k k}{10e^k}$$

4. (7 points) Find the radius of convergence and interval of convergence for the following power series:

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}(x-6)^k}{4^k \cdot k}$$

5. (8 points) Compute the Taylor series for

$$f(x) = \frac{1}{(x+3)^2}$$

centered at $a = -2$ **using the definition of the Taylor series**. Write the series using summation notation. (You **do not** need to find the radius of convergence or the interval of convergence.) Simplify your answer.

6. (8 points) Consider the following parametric equations:
$x = e^t$ and $y = 4 - e^{2t}$ ; $-\infty < t \leq \ln 2$

(a) Eliminate the parameter $t$ to obtain an equation in $x$ and $y$.

(b) Sketch the curve and indicate the positive orientation.

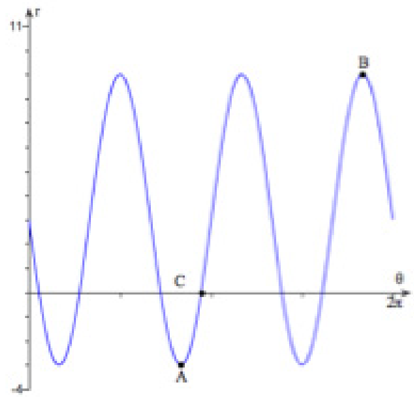
7. (6 points) Sketch each of the following sets of points in the polar plane. (Use separate sketches for each set.)

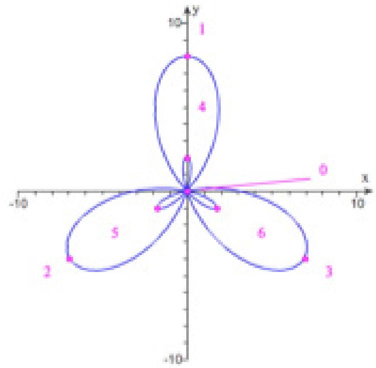
(a) $\{(r, \theta) : |r| \geq 2 \text{ and } \frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}\}$

(b) $\{(r, \theta) : 0 < r \leq 3 \text{ and } \frac{2\pi}{3} \leq \theta \leq \frac{5\pi}{3}\}$

8. (8 points) Polar Curves

(a) A Cartesian and a polar graph are given. Identify the points on the polar graph that correspond to the points shown on the Cartesian graph.



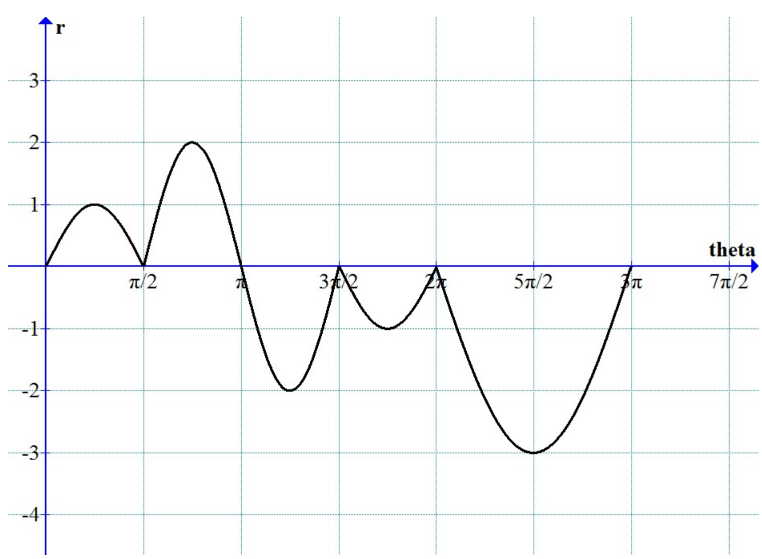



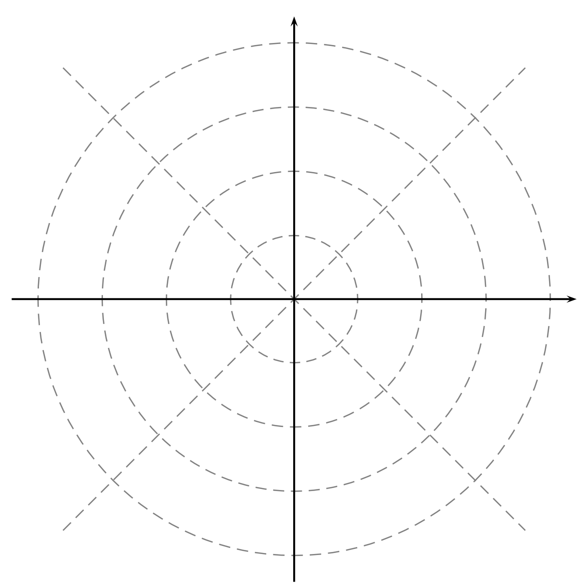

i. Point A on the Cartesian graph corresponds to point ___on the polar graph.

ii. Point C on the Cartesian graph corresponds to point ___on the polar graph.

iii. Point B on the Cartesian graph corresponds to point ___on the polar graph.

(b) The following is the graph of a polar curve drawn in the Cartesian plane. Draw the curve in the polar plane next to it..



9. (12 points) Complex numbers

(a) i. Write $(-\sqrt{3}+i)$ in polar form.

ii. Find $(-\sqrt{3}+i)^6$ and write your answer in rectangular form.

(b) i. Find the cube roots of $z=-8$.

ii. Plot the roots in the complex plane.

**You may use any of the following for the rest of the exam.**

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^k + \cdots = \sum_{k=0}^{\infty} x^k, \quad \text{for } |x| < 1$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + \frac{x^k}{k} + \cdots = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k}, \quad \text{for } -1 < x \leq 1$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^k}{k!} + \cdots = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \quad \text{for } |x| < \infty$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{(-1)^{k+1} x^{2k+1}}{(2k+1)!} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, \quad \text{for } |x| < \infty$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{(-1)^k x^{2k}}{(2k)!} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}, \quad \text{for } |x| < \infty$$

10. (5 points) Evaluate the following limit using series:

$$\lim_{x \to 0} \frac{2\cos 2x - 2 + 4x^2}{2x^4}$$

11. (6 points) Identify each of the functions represented by the following power series.

(a) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^{k+1} x^{6k+3}}{(2k+1)!}$

(b) $\displaystyle\sum_{k=1}^{\infty} 2^k \cdot k \cdot x^{k-1}$

**You may use any of the following for the rest of the exam.**

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^k + \cdots = \sum_{k=0}^{\infty} x^k, \quad \text{for } |x| < 1$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + \frac{x^k}{k} + \cdots = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k}, \quad \text{for } -1 < x \le 1$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^k}{k!} + \cdots = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \quad \text{for } |x| < \infty$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{(-1)^{k+1} x^{2k+1}}{(2k+1)!} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, \quad \text{for } |x| < \infty$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{(-1)^k x^{2k}}{(2k)!} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}, \quad \text{for } |x| < \infty$$

12. (8 points) Use a Taylor series to approximate the following definite integral. Retain as many terms as needed to ensure the error is less than $\frac{1}{5000}$. Justify and simplify your answer.

$$\int_0^{0.1} \frac{\ln(1+x)}{x}\, dx$$

Math 34 Final Exam May 2, 2014

Name ______________________________

Circle your section:

32-01 Hao Liang TThF 8:30-9:20

32-02 Mary Glaser TThF 12-12:50 SECRET 4 digit code number: ________

32-03 Gail Kaufmann TTHF 12-12:50

32-04 Thomas Benson TTh1:30-2:20,F2:30-3:20

I pledge that I have neither given nor received assistance on this exam.

Signature ______________________________