1. (9 points)

(a) Find the two complex numbers $z$ which are solutions to the equation

$$2z^2 - 4z + 3 = 0.$$ 

(b) Express the complex number $z = e^{1+5ni/3}$ in the form $z = a + bi$ for real numbers $a, b$.

c) Express $z = \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^{60}$ in the form $z = a + bi$ for real numbers $a, b$.

2. (24 points) Evaluate the following integrals:

(a) $\int \frac{x^2 + 8x + 7}{(x - 1)(x^2 + 6x + 9)} \, dx$

(b) $\int \frac{1}{(1 + 4x^2)^{3/2}} \, dx$

(c) $\int x \ln x \, dx$

3. (8 points) Using the method of cylindrical shells, set up, but do not evaluate, an integral or integrals for the volume generated by revolving the region bounded by the curves $y = \ln x$, $y = 0$, $x = e$ about the $y$-axis. Be sure to draw a picture of the region you are revolving.

4. (8 points) Without using series solutions, solve the initial value problem:

$$\frac{dy}{dx} = \frac{2x + \sec^2 x}{2y}, \quad y(0) = -5.$$ 

Express your answer as $y = f(x)$, i.e. solve for $y$ explicitly.

Exam continues on reverse.
5. (12 points)
(a) (5 pts) Determine whether the series \( \sum_{n=1}^{\infty} \frac{(\sin^2 n) \sqrt{n}}{n^3 + 1} \) converges or diverges.
(b) (7 pts) Find the radius of convergence \( R \) and the interval of convergence \( I \) of the power series
\[
\sum_{n=2}^{\infty} \frac{2^n x^n}{\ln n}.
\]

6. (5 points) A sequence of numbers is given by the following recursion relation:
\[
a_{n+1} = \frac{a_{n-1}}{n} \quad \text{for } n \geq 1, \quad a_0 = 0, \quad a_1 = 3.
\]
Solve for \( a_n \).

7. (9 points) You may use the fact that \( e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \) for all \( x \in \mathbb{R} \) in answering the following questions.
(a) (3 pts) Choose one: The absolute value of the error made in approximating \( e^{-2} \) by the 4th degree Taylor polynomial about 0 evaluated at \( x = -2 \), \( T_4(-2) \), is \( \begin{cases} \text{larger than} & \frac{1}{3} \\ \text{less than} & \frac{1}{3} \\ \text{equal to} & \frac{1}{3} \end{cases} \).
In justifying your answer, you may assume the series for \( e^{-2} \) satisfies the conditions of the Alternating Series Theorem.
(b) (6 pts)
(i) Use a change of indices to help you evaluate the following infinite series:
\[
\sum_{n=2}^{\infty} \frac{(-1)^n 2^n}{(n-2)!}.
\]
(ii) Determine the function equal to the following power series:
\[
\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2^n n!}.
\]

8. (10 points)
Consider the curve given by the parametric equations
\[
C = \{(x, y) : x = \sin t, \ y = e^{\sin t}\}.
\]
(a) (3 pts) Eliminate the parameter \( t \) to find a Cartesian equation of the curve.
(b) (7 pts) Sketch the curve \( C \) and indicate with arrows the path in which it is traversed as \( t \) increases from \( t = -\pi/2 \) to \( t = \pi \). Be sure to label the starting and ending points.
9. (7 points) (a) (3 pts) Below are graphs of the two polar curves

\[ C_1 : r = \sin \theta, \ 0 \leq \theta \leq \pi \quad \text{and} \quad C_2 : r = \sin(2\theta), \ 0 \leq \theta \leq \pi/2. \]

Copy the graphs in your blue book. Label the curves and indicate the polar coordinates of the points at which they intersect.

(b) (4 pts) Set-up, but do not evaluate, an integral or integrals for the area which is inside \( C_2 \) but outside \( C_1 \).

10. (8 points) Do this problem on a new page.

(a) (2 pt) Identify the polar curve: \( r \sin \theta = 1 \) by changing the equation to \((x, y)\) Cartesian coordinates.

(b) (6 pts) Below is the \((\theta, r)\) Cartesian graph of a polar curve. The portion for \( \pi/4 \leq \theta \leq \pi/2 \) corresponds to a portion of the curve you identified in part (a). (The remainder of the graph corresponds to a different polar curve.)

Using this graph together with what you know from part (a), graph the polar curve in \((r, \theta)\) polar coordinates. Make the drawing large with 4 bluebook squares per 1 unit. Label all points at which your polar curve intersects the Cartesian axes (i.e. the \( x - y \) axes); use \((r, \theta)\)-coordinates for your labels.

END OF EXAM.