EXAM 2 Math 34 (Calc II), Spring 2013

NAME, SECTION

No notes or devices with calculator functionality. Turn your phone off!

Some questions have boxes provided for you to write your answers. **Only the work in the boxes will be graded.** Please use this sheet for scratch work or ask for scratch paper if needed. (It's fine to have some small amounts of scratch work in the margins.)

You must evaluate standard trig values, but you do not need to simplify your answers otherwise.

Except for true/false and multiple choice, you must show work to receive full credit.

You must verify the hypotheses for any convergence test that you use.

If you claim that an inequality holds, you must justify that claim except if you judge it to be obvious. For example, for $n = 1, 2, 3, \ldots$, the inequality $n < 3n + 2$ is obvious, while $2(n + 10)! \leq (2n)!$ is not.

I pledge that I have neither given nor received assistance on this test.

(signature)

(1) (12 pts) **True or false.** Indicate your answer by circling T or F.

(a) If a series $\sum_{k=1}^{\infty} a_k$ sums to 15, then the sequence $\{a_k\}$ converges to 15. T F

(b) If $\sum_{k=1}^{\infty} a_k$ converges, and $0 < a_k \leq b_k$ for all $k \geq 1$, then $\sum_{k=1}^{\infty} b_k$ converges. T F

(c) The sequence $\{\sin n\}$ converges because it is bounded. T F

(d) For every p-series $\sum_{k=1}^{\infty} \frac{1}{k^p}$, the ratio test is inconclusive. T F

(e) The sequence $\left\{ \frac{(-1)^n + 3}{\sqrt{n}} \right\}$ converges to 0 by the Squeeze Theorem. T F

(f) $-\frac{3}{5} + \frac{9}{25} - \frac{27}{125} + \frac{81}{625} - \cdots = -\frac{3}{8}$ T F

(2) (8 pts) Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!}$, and suppose you know it converges.

Estimate the sum of the series with an absolute error less than $\frac{1}{500}$.

(3) (15 pts) **Multiple choice.** Circle ALL correct answers.

(a) Suppose a power series $\sum_{n=0}^{\infty} c_n(x+1)^n$ has radius of convergence 4.

Then the series ...

(i) converges at $x = -2$;

(ii) converges at $x = 0$;

(iii) converges at $x = 2$;

(iv) converges at $x = 4$.

(b) The series $\sum_{n=1}^{\infty} \frac{n!}{2^n}$ ...

(i) converges by the integral test;

(ii) converges by the ratio test;

(iii) diverges by the ratio test;

(iv) is a divergent geometric series;

(v) converges by the divergence test.

(c) A sequence has the recursive formula $a_{n+1} = a_n + 3$ and initial value $a_0 = 2$.

An explicit formula for the nth term is ...

(i) $a_n = n + 2, \quad n \geq 0$;

(ii) $a_n = 2n + 2, \quad n \geq 0$;

(iii) $a_n = 3n + 2, \quad n \geq 0$;

(iv) $a_n = 3n + 1, \quad n \geq 0$.

(d) The series $\sum_{k=1}^{\infty} \left(\frac{5}{k+2}\right)^k$ ...

(i) diverges by the root test;

(ii) converges by the root test;

(iii) makes the root test inconclusive;

(iv) is a geometric series;

(v) diverges by the divergence test.

(e) The sequence $\left\{\left(1 - \frac{2}{k}\right)^k\right\}$ ...

(i) converges to 0;

(ii) converges to $1/e^2$;

(iii) converges to 1;

(iv) converges to e^2;

(v) diverges.

(4) (15 pts) Determine whether each of the following series converges or diverges. If the series converges, find its sum.

(a) $\displaystyle\sum_{k=1}^{\infty} \ln\left(2+\frac{1}{k}\right)$

(b) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{1}{k+3}-\frac{1}{k+2}\right)$

(5) (16 pts) Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent.

(a) $\displaystyle\sum_{k=1}^{\infty} (-1)^k \cdot \frac{k+1}{k^2}$

(b) $\displaystyle\sum_{k=1}^{\infty} k \cdot \left(-\frac{1}{2}\right)^k$

(6) (8 pts) Recall that $\dfrac{1}{1-x} = \displaystyle\sum_{n=0}^{\infty} x^n$ for $|x| < 1$.

Find a power series representation for $f(x) = \dfrac{x}{1 + \frac{x^2}{4}}$ and give its radius of convergence.

(7) (8 pts) Find the radius of convergence and interval of convergence of the power series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k \cdot (x-3)^k}{k \cdot 2^k}$.

(8) (8 pts) Starting with the power series $f(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x-2)^k}{3^k}$ ...

(a) find a power series representation for $f''(x)$.

(b) find a power series representation for $\int f(x)\,dx$.

(9) (10 pts)

(a) For $f(x) = \sin(2x)$, find the fourth-degree Taylor polynomial $p_4(x)$ centered at $a = \frac{\pi}{6}$.

(b) What is the quadratic polynomial that best approximates $\sin(2x)$ near $x = \frac{\pi}{6}$?

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	TOTAL
[12]	[8]	[15]	[15]	[16]	[8]	[8]	[8]	[10]	**[100]**