

No books, notes or calculators are allowed. Cross out what you do not want us to grade. You must show all your work in order to receive full credit. You will be expected to simplify any standard trigonometric values, e.g.  $\sin(\frac{\pi}{2})$  or  $\arctan(1)$ . Please write neatly. You are required to sign your exam book. With your signature, you pledge that you have neither given or received assistance on this exam. **IF YOU DO NOT TURN IN YOUR EXAM WHEN THE PROCTOR ANNOUNCES THAT THE EXAM IS OVER THE GRADE YOU RECEIVE WILL BE 90% OF YOUR COMPUTED SCORE.**

1. (14 points) True or False. Please list the letters from (a) to (g) on the first page of your blue book and put your answers to this problem there. Please write the whole word "true" or "false".
  - (a) A conditionally convergent series converges.
  - (b) If  $\sum_{n=1000}^{\infty} a_n$  converges,  $\sum_{n=1}^{\infty} a_n$  converges.
  - (c)  $\sum_{n=1}^{\infty} 1$  converges.
  - (d) A bounded sequence converges.
  - (e) The series  $\sum_{n=1}^{\infty} a_n$  converges if  $\lim_{n \rightarrow \infty} a_n = 0$ .
  - (f) The radius of convergence of  $\sum_{n=0}^{\infty} (\frac{x}{3})^n$  is  $\frac{1}{3}$ .
  - (g) If  $\sum_{n=0}^{\infty} c_n x^n$  converges when  $x = 4$ , it converges when  $x = 2$ .
  
2. (10 points) Determine if the sequence is convergent or divergent. If the sequence converges find its limit. Show work.

$$(a) a_n = (-1)^n \left( \frac{14n^2 + 11}{2n^2 + 3n} \right)$$

$$(b) a_n = \frac{(2(n+1))!}{(n^2) \cdot (2n)!}$$

3. (10 points)

(a) Write out and *simplify* the  $n^{\text{th}}$  partial sum  $s_n$  for the following series:

$$\sum_{n=1}^{\infty} \left( \cos\left(\frac{\pi}{n}\right) - \cos\left(\frac{\pi}{n+1}\right) \right)$$

(b) Based on your work above, find the sum of the series or determine that the series diverges.

(please turn over)

4. (16 points) Determine whether each of the following series converges or diverges. Justify your answer. State and check hypotheses of any test, rules or theorems you use. If the series converges, find its sum.

$$(a) \sum_{n=1}^{\infty} \frac{3^{n+1}}{\pi^{n-1}}$$

$$(b) \sum_{n=1}^{\infty} \frac{e^n}{n^2}$$

Determine whether each of the following series diverges, is absolutely convergent, or conditionally convergent. Justify your answer. State and check hypotheses of any test, rules or theorems you use. (8 points each)

5. 
$$\sum_{n=1}^{\infty} (-1)^n \frac{\arctan(n)}{n^2 + n}$$

6. 
$$\sum_{n=2}^{\infty} (-1)^n \frac{n}{\ln(n)}$$

7. (10 points)

(a) Find the radius (*not* the interval) of convergence for the following power series:

$$\sum_{n=1}^{\infty} (-1)^n \frac{(x+4)^n}{5^n \cdot n^2}$$

(b) Based on the radius of convergence you found above, write down simplified versions of the series, if any, that would need to be tested to determine the interval of convergence of this series. **Do not test them, just list them.**

8. (14 points) Use the fact that  $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$  for  $|x| < 1$  to find each of the following :

(a) A power series representation of  $\frac{2x^2}{4+3x}$  and its radius of convergence.

(b) A power series representation of  $\ln(1+x)$  and its radius of convergence.

9. (10 points) Consider 
$$\sum_{n=1}^{\infty} (-1)^n \frac{n+6}{n^4+n}$$

(a) Show this series passes the Alternating Series Test and hence converges to a sum  $s$ .

(b) Find the smallest  $n$  such that  $|s - s_n|$  is less than  $\frac{1}{25}$ .