Tufts University

Department of Mathematics

Math 12, Test 2

Monday, November 10, 2008 **12–1:20 p.m.**

No calculators, notes, scratch paper or books are allowed. Except where indicated, you must show all your work in your blue book in order to receive full credit. A correct answer with no work might be given zero points. Cross out any work you do not want graded. Sign your exam book, indicating that you have neither given nor received help during this exam. Any violations will be reported to the appropriate dean, and will result in an F for the course.

There are ten problems on this exam, each worth 10 points.

1 Determine whether each of the series converges or diverges. If it converges, find the sum.

$$a)\ \sum_{n=1}^{\infty} \frac{1}{n(n+1)} \qquad b)\ \sum_{n=0}^{\infty} (-1)^n \frac{3^{2n}}{4^{n-1}}$$

Determine whether each of the series converges or diverges.

2 $$\sum_{n=0}^{\infty} \frac{3\sin^2(n) + 10\cos^4(n)}{5^n}$$

3 $$\sum_{n=3}^{\infty} \frac{1}{(n-1)\ln(n-1)}$$

4 $$\sum_{n=1}^{\infty} \frac{4n^2}{2^{n+1}}$$

5 $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+1}$$

6 Determine whether the series converges conditionally, converges absolutely, or diverges.

$$\sum_{n=0}^{\infty} (-1)^n \frac{\sqrt[3]{n+5}}{n^2+2}$$

Find the radius of convergence and the interval of convergence for the following power series.

7 $$\sum_{n=1}^{\infty} 5 \cdot 2^n (x+3)^n$$

8 $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{4^n \sqrt{n}}$$

9 *a*) Using the identity

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \qquad |x| < 1,$$

find a power series expansion for the function

$$f(x) = \frac{1}{1+x^3}.$$

b) Estimate the following definite integral to within an accuracy of 1/100. You do not need to show that the definite integral of your answer from part a) satisfies the alternating series test.

$$\int_0^{\frac{1}{2}} \frac{1}{1+x^3}\, dx$$

10 Find the Taylor series centered at 1 for the function

$$f(x) = \frac{2}{x^3}.$$

Assume that $f(x)$ has a power series expansion. Do not show that $R_n(x) \to 0$.