

Tufts University
Department of Mathematics
Math 12, Test 1

Monday, October 19, 2009

12:00 p.m.-1:20 p.m.

No calculators, notes, scratch paper or books are allowed. You must show all your work in your blue book in order to receive full credit. A correct answer with no work might be given zero points. Cross out any work you do not want graded. Sign your exam book, indicating that you have neither given nor received help during this exam. Any violations will be reported to the appropriate dean, and will result in an F for the course.

For problems 1 to 4, evaluate the indefinite integrals.

1. (12 points): $\int x^2 e^{(2x)} dx$

2. (12 points): $\int \frac{5x^2 - x - 2}{(x^2 + 1)(x - 1)} dx$

3. (12 points): $\int \frac{1}{x} \tan^3(\ln(x)) \sec^3(\ln(x)) dx$

4. (12 points): $\int \frac{\sqrt{x^2 - 4}}{x^3} dx$

5. (12 points): True or False: The integral $\int_1^{\infty} \frac{x}{x^4 + 12} dx$ is convergent. Explain your answer.

6. (10 points): Set up a definite integral for the volume of the solid obtained by rotating the bounded region between the graphs of $y = x^2$ and $y = 2 - x$ about the line $y = 4$. Use the methods of disks or washers as discussed in class. Draw a picture of the region which is rotated.
DO NOT EVALUATE THE DEFINITE INTEGRAL.

7. (10 points): Find the area of the bounded region enclosed by the curves $y = 2 \sin(x)$, $y = 2 \cos(x)$, $x = 0$ and $x = \pi/2$.

The exam continues on the back of this page.

8. (10 points): A population of squirrels in a park increases naturally according to the law of natural growth. However, on average, 10 leave the park every year. As our story begins, at $t = 0$ years, there are 20 squirrels. If $P(t)$ is the population of squirrels in the park at t years, then $P(t)$ satisfies the differential equation

$$\frac{dP}{dt} = 5P - 10. \quad (1)$$

- (a) Solve the differential equation (1) with the initial condition $P(0) = 20$ as given in the problem. Show all your work.
- (b) When are there 38 squirrels? You may leave your answers in terms of logs and exponents.
9. (10 points): Determine if the following sequences are convergent or divergent. If a sequence converges, find its limit (as a number).

(a) $a_n = \arctan\left(\frac{n}{n+1}\right)$

(b) $a_n = (-1)^n \left(\frac{n}{n+1}\right)$