Math 11 - Fall 2011 - Final Exam - Solutions

(1) (a) F, (b) [illegible], (c) F, (d) T, (e) F

(2) Need: $h + 2w + 2d = 54$ but square side means $h = d$ so we have: $\boxed{2w + 3d = 54}$. Volume is given by $V = wd^2$

$2w + 3d = 54 \rightarrow 2w = 54 - 3d \rightarrow w = 27 - \frac{3}{2}d$

$V(d) = (27 - \frac{3}{2}d) \cdot d^2 = 27d^2 - \frac{3}{2}d^3$

$V'(d) = 54d - \frac{9}{2}d^2$. $V'(d) = 0$ when $\frac{9d}{2}(12 - d) = 0 \rightarrow d = 12$, $\cancel{d=0}$

(notice $d \le 18$ since $2w + 3d = 54$)

$V(d)$ maximum when $d = 12$

dimensions: $2w + 3(12) = 54$
$2w = 54 - 36$
$w = 9$

$\boxed{9\text{ in} \times 12\text{ in} \times 12\text{ in}}$

(3) $f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h}$

$= \lim_{h\to 0} \frac{\frac{2-(2+h)}{2(2+h)}}{h} = \lim_{h\to 0} \frac{-h}{2h(2+h)} = \boxed{-\frac{1}{4}}$

(b) $f'(x) = -\frac{1}{x^2}$; $f'(1) = -1$

$f(1) = 1$

$\boxed{y - 1 = -1(x - 1)}$

(4)(a) $F'(x) = [4(x^3)^2 + 2] \cdot 3x^2$

b) $F'(x) = \frac{1}{1 + [\ln(x^2 - 4x)]^2} \cdot \frac{1}{(x^2 - 4x)} \cdot (2x - 4)$

5)(a) $\lim_{x\to\infty} \frac{3x+1}{\sqrt{x^3 + 2x + 5}} = \lim_{x\to\infty} \frac{3x}{\sqrt{x^3}} = \lim_{x\to\infty} \frac{3x}{x^{3/2}}$

$= \lim_{x\to\infty} \frac{3}{x^{1/2}} = \boxed{0}$

(5)(b)

$\lim_{x\to 1} \frac{x}{x-1} - \frac{1}{\ln x}$

$= \lim_{x\to 1} \frac{x \cdot \ln x - (x-1)}{(x-1) \cdot \ln x} = \lim_{x\to 1} \frac{x \cdot \ln x - x + 1}{(x-1) \cdot \ln x}$

$\overset{L}{\underset{0/0}{=}} \lim_{x\to 1} \frac{x \cdot \frac{1}{x} + 1 \cdot \ln x - 1}{(x-1)\frac{1}{x} + \ln x} = \lim_{x\to 1} \frac{\ln x}{1 - \frac{1}{x} + \ln x}$

$\overset{L}{\underset{0/0}{=}} \lim_{x\to 1} \frac{\frac{1}{x}}{\frac{1}{x^2} + \frac{1}{x} \cdot \frac{x}{x}} = \lim_{x\to 1} \frac{\frac{1}{x}}{\frac{1+x}{x^2}}$

$= \lim_{x\to 1} \frac{x^2}{x(x+1)} = \boxed{\frac{1}{2}}$

(6) (a) horizontal at $y=1$, vertical at $x=0$

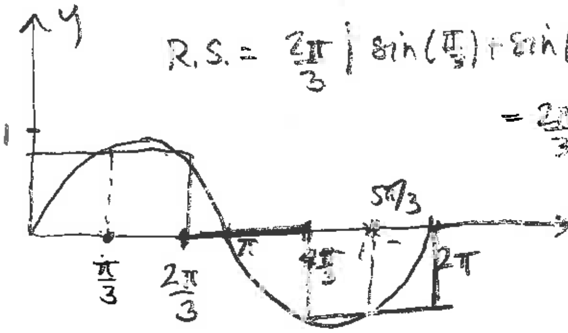
(b) $(-2,0)$

(c) Local min at $x=-2$

(d) $(-3,0) \cup (0,\infty)$

(e) IP at $\boxed{x=-3}$

(7) (a)



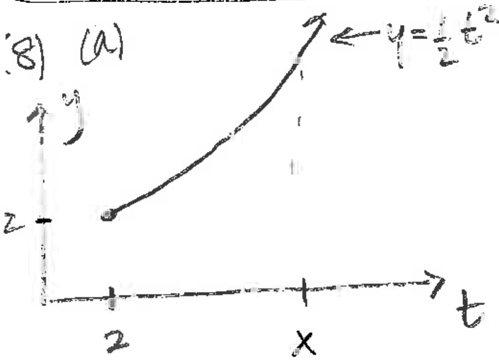
$\Delta x = \frac{2\pi}{3}$

$R.S. = \frac{2\pi}{3}\left[\sin\left(\frac{\pi}{3}\right)+\sin(\pi)+\sin\left(\frac{5\pi}{3}\right)\right]$

$= \frac{2\pi}{3}\left[\frac{\sqrt{3}}{2}+0-\frac{\sqrt{3}}{2}\right] = \boxed{0}$

(b) $\int_0^{2\pi} \sin x\,dx = 0$ using net area interpretation

(8) (a)



(b) $A(4) = \int_2^4 \frac{1}{2}t^2\,dt = \frac{1}{2}\frac{t^3}{3}\Big]_2^4 = \frac{4^3}{6}-\frac{8}{6} = \frac{56}{6} = \boxed{\frac{28}{3}}$

(c) $A(x) = \int_2^x \frac{1}{2}t^2\,dt = \left(\frac{1}{6}t^3\right)\Big|_2^x = \boxed{\frac{x^3}{6}-\frac{8}{6}}$

(d) $A'(x) = \frac{3x^2}{6} = \frac{x^2}{2}$; yes this matches $\frac{d}{dx}\left(\int_2^x \frac{1}{2}t^2\,dt\right) = \frac{1}{2}x^2$

(9) (a) $\int \frac{m^3-m+1}{m^2}\,dm = \int m-\frac{1}{m}+m^{-2}\,dm = \boxed{\frac{m^2}{2}-\ln|m|+\frac{m^{-1}}{-1}+C}$

(b) $\int 3\sin x\cos x\,dx = \int 3u\,du = \frac{3u^2}{2}+C = \boxed{\frac{3\sin^2 x}{2}+C}$

$u=\sin x$

$du=\cos x\,dx$

(c) $\int_{1/2}^{3} \frac{1}{\sqrt{3+2x}}\,dx = \frac{1}{2}\int_{3+1}^{3+2(3)} \frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int_4^9 u^{-1/2}\,du = \frac{\cancel{2}u^{1/2}}{\cancel{2}}\Big]_4^9 = \sqrt{9}-\sqrt{4}$

$= 3-2 = \boxed{1}$

$u=3+2x$

$du = 2\,dx$