Math 11 Exam 2 Solutions

Solutions : Spring 2011

1 (a) $4/5 = \text{(iii)}$

(b) $\pi/4 = \text{(i)}$

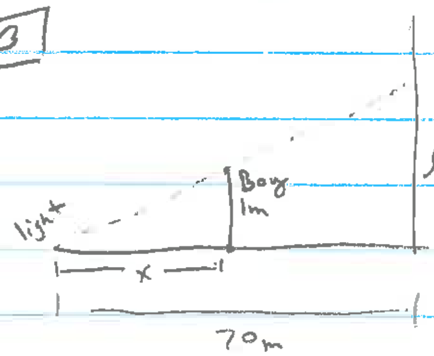
2 (a) $\dfrac{dy}{dx} = \dfrac{1}{1+(\sqrt{x+1})^2}\,\dfrac{1}{2}(x+1)^{-1/2} = \dfrac{1}{2(2+x)(x+1)^{1/2}}$

(b) $\ln y = 2x\ln(\cos x) \Rightarrow \dfrac{y'}{y} = 2\cdot\ln(\cos x) + 2x\cdot\dfrac{1}{\cos x}\cdot(-\sin x)$

$\Rightarrow y' = \cos(x)^{2x}\cdot\left[2\ln(\cos(x)) - \dfrac{2x\sin x}{\cos x}\right]$

(c) $\dfrac{dy}{dx} = \dfrac{\frac{1}{x}\cdot x^2 - 2x\ln(x)}{x^4}\left(= \dfrac{x - 2x\ln(x)}{x^4} = \dfrac{1-2\ln x}{x^3}\right)$

3



l = length of shadow at time t

x = distance in m of boy to light

Similar triangles: $\dfrac{l}{20} = \dfrac{1}{x} \Rightarrow l = \dfrac{20}{x}$

Know: $\dfrac{dx}{dt} = -1$ m/sec

$\dfrac{dl}{dt} = \dfrac{-20}{x^2}\left(\dfrac{dx}{dt}\right)$ when $x = 10$ m $\left.\dfrac{dl}{dt}\right|_{x=1} = \dfrac{-20}{(10)^2}\cdot(-1) = \dfrac{20}{100}$ m/sec

$= \boxed{\frac{1}{5} \text{ m/sec}}$

$\boxed{4}$ differentiating implicitly gives

$$\frac{1}{2}(xy)^{-1/2}\cdot(y+xy') = 2xy + x^2y'$$

So when $(x,y)=(1,1)$, we have

$$\frac{1}{2}(1)^{-1/2}\cdot(1+y') = 2+1^2\cdot y' \Rightarrow 1+y' = 4+2y'$$

$$\Rightarrow \boxed{y'=-3}$$

so eq for tangent line is $\boxed{y-1=-3(x-1)}$

$\boxed{5}$ (a) since $f'>0$ on $(-1,1)$ and $f'<0$ on $(1,2)$,

$x=1$ is the only critical number, and

f has a local maximum at $x=1$

(b) f' is increasing on $(-1,0)$ and $(3/2, 2)$

ie $f''>0$ on " "

so graph of f is concave up on " "

(c) - $f''>0$ on $(-1,0)$ and $f''<0$ on $(0,3/2)$

so f has inflection point at $x=0$

- $f''<0$ on $(0,3/2)$ and $f''>0$ on $(3/2,2)$

so f has inflection point at $x=3/2$

6 $\lim_{x\to 0} \dfrac{e^{2x}-1-2x}{x^2} \underset{\text{Rule}}{\overset{\text{l'Hopital's}}{=}} \lim_{x\to 0} \dfrac{2e^{2x}-2}{2x} \overset{\text{l'Hop.}}{=} \lim_{x\to 0} \dfrac{4e^{2x}}{2} = 2$

$e^0-1-0=0$ as $x\to 0$; $0^2=0$ as $x\to 0$

$2e^0-2=0$ as $x\to 0$; $2x=0$ as $x\to 0$

7 $f'(x) = 3x^2-3 \overset{?}{=} 0$ when $x^2=1$ ie $\boxed{x=\pm 1}$

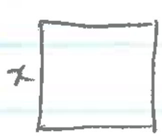
(a) So only critical point in $(-3,0)$ is $\boxed{x=-1}$.
To find abs. max / min, compute the following values

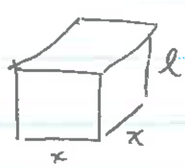
X	-3	-1	0
$f(x)$	$-27+9+1$ $=-17$	$-1+3+1=3$	1

(b) abs max value is 3, & $f(-1)=3$

(c) abs min value is -17, & $f(-3)=-17$

8 Base





$V = x^2 l$

Constraint: $125 = x^2 l \Rightarrow l = \dfrac{125}{x^2}$

Cost: if c_0 = cost of material for base, $\frac{1}{2}c_0$ = cost of material for sides.

Then $C = c_0 x^2 + \frac{1}{2}c_0 \cdot 4 \cdot x l$

$\Rightarrow C = c_0\left(x^2+2xl\right) = c_0\left(x^2+\dfrac{2\cdot 125}{x}\right)$

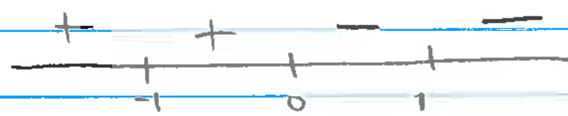
Now, $C' = c_0\left(2x - \dfrac{2\cdot 125}{x^2}\right) \overset{?}{=} 0 \Rightarrow x^3 = 125 \Rightarrow \boxed{x = 5 \text{ cm}}$

& $C'' = c_0\left(2+\dfrac{4\cdot 125}{x^3}\right)\Big|_{x=5} > 0 \Rightarrow C$ has $\underline{\text{abs. min}}$ at $x=5$ (for x in $(0,\infty)$)

$\boxed{9}$ (a) domain $= (-\infty,-1), (-1,1), (1,\infty)$

since f has VA's at $x=1$ & $x=-1$.

(b) sign chart for f': Note critical numbers are $0, -1, 1$:



$\therefore$ intervals of increase are $(-\infty,-1)$ and $(-1,0)$

(c) and intervals of decrease are $(0,1)$ and $(1,\infty)$

(d) f has a local maximum at $x=0$ (by first der. test together with the sign chart)

(e) no local min

(f) sign chart for f'' $\quad 12x^2+4 \neq 0$ for any x

$\therefore$ f'' can only change sign for $x=\pm 1$

x^2-1 ; same sign as f''

$\therefore$ Concave up on $(-\infty,-1)$ and $(1,\infty)$

(g) and concave down on $(-1,1)$

(h) no inflection points (since f undefined at $x=\pm 1$).