Math 32 - Spring 2013 - Exam I - Solutions.

(1) (a) The <u>average velocity</u> of the object over the time interval $[a,b]$.

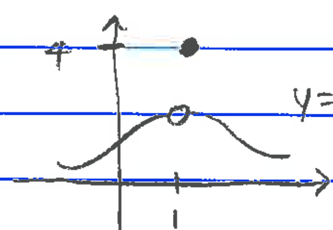
(b) $h'(2) = f'(g(2))\cdot g'(2) = f'(7)\cdot g'(2)$ so we need $f'(7)$ to find $h'(2)$ (since $g'(2)=6$ given).

(c) OMITTED since we forgot to say f and g are continuous fncs.

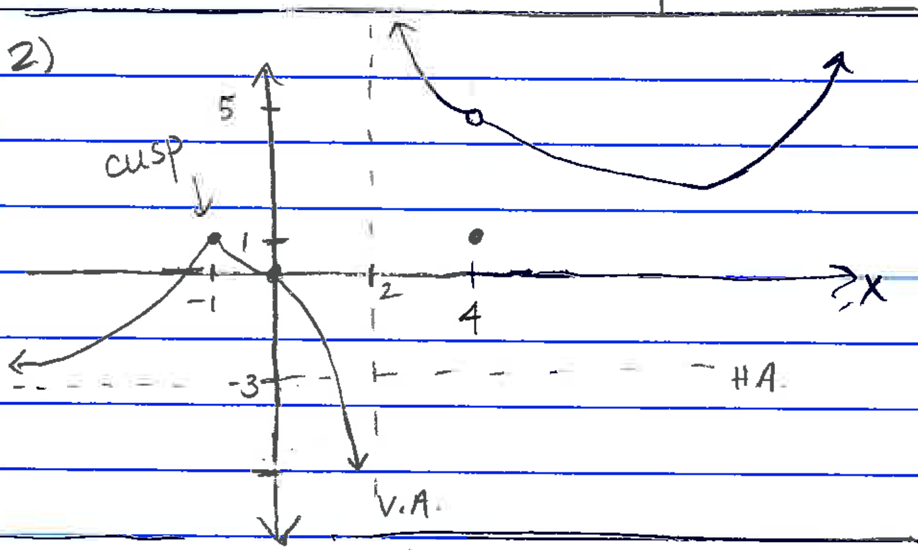
(d) $\lim_{x\to\infty} e^{-2x}+3 = 3$ since $e^{-2x}\to 0$ as $x\to\infty$. so $\boxed{y=3}$ a horiz. asymptote.

$\lim_{x\to-\infty} e^{-2x}+3 = \infty$ so $y=3$ the only one

(e)



(2)



(3) (a) $\lim_{x\to 0^+} \frac{\tan x}{x} = \lim_{x\to 0^+} \frac{\sin x}{\cos x}\cdot\frac{1}{x}$

$= \lim_{x\to 0^+} \frac{\sin x}{x}\cdot\frac{1}{\cos x} = 1\cdot 1 = \boxed{1.}$

↑ limit laws

(b) $\lim_{x\to\frac{\pi}{2}^-} \frac{\tan x}{x} = \frac{+\infty}{+} = \infty$ $\boxed{\text{So yes } x=\frac{\pi}{2} \text{ a V.A.}}$

(5)

(a) $\lim_{x\to 3^+} \frac{x}{\sqrt{x-3}} = \frac{+}{+}\infty = +\infty$

(b) Since $-1 \le \sin x \le 1$, we have $1 \le 2+\sin x \le 3 \rightarrow$

$\frac{1}{\sqrt{x}} \le \frac{2+\sin x}{\sqrt{x}} \le \frac{3}{\sqrt{x}}$.

Then $\lim_{x\to\infty} \frac{1}{\sqrt{x}} = 0 = \lim_{x\to\infty}\frac{3}{\sqrt{x}}$

so $\lim_{x\to\infty} \frac{2+\sin x}{\sqrt{x}} = 0$

by Squeeze Theorem.

(4) (a) $2(x^3-4x^2)=0$

$2x^2(x-4)=0 \rightarrow x=0,4$ $\boxed{\text{So domain is all } x \neq 0,4.}$

(b) $\lim\limits_{x\to 4} f(x) = \lim\limits_{x\to 4} \dfrac{x(x^2+2x-24)}{2x^2(x-4)} = \lim\limits_{x\to 4} \dfrac{x\cancel{(x-4)}(x+6)}{2x^2\cancel{(x-4)}} = \dfrac{10}{8} = \boxed{\dfrac{5}{4}}$

(c) $\lim\limits_{x\to +\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{x^3}{2x^3} = \dfrac{1}{2} = \lim\limits_{x\to -\infty} f(x)$ $\boxed{\text{so } y=\tfrac{1}{2} \text{ is the only H.A.}}$

(6) (a) $\boxed{\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = f'(x) \text{ OR } \lim\limits_{x\to a} \dfrac{f(x)-f(a)}{x-a} = f'(a)}$

(b) $f'(x) = \lim\limits_{h\to 0} \dfrac{(x+h)e^{x+h} - xe^x}{h} = \lim\limits_{h\to 0} \dfrac{xe^{x+h} + he^{x+h} - xe^x}{h}$

$= \lim\limits_{h\to 0} \dfrac{\cancel{h}e^{x+h}}{\cancel{h}} + \dfrac{xe^x \cdot e^h - xe^x}{h} = \lim\limits_{h\to 0} e^{x+h} + xe^x\left(\dfrac{e^h-1}{h}\right)$

$= \underbrace{\lim\limits_{h\to 0} e^{x+h}}_{=e^x} + xe^x \underbrace{\left(\lim\limits_{h\to 0} \dfrac{e^h-1}{h}\right)}_{=1} = e^x + xe^x$

(c) $f'(x) = x\cdot e^x + 1\cdot e^x = xe^x + e^x$ ✓

(d) $f'(0) = 0\cdot e^0 + e^0 = \boxed{1}$

(7) $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} x^3 = 0$; $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} x^2 = 0$

So $\lim\limits_{x\to 0} f(x) = 0$. But $f(0)=0$ too, so $\lim\limits_{x\to 0} f(x) = f(0)$

which means f is cont. at $x=0$

(b) $\lim\limits_{x\to 0^+} \dfrac{f(x)-f(0)}{x-0} = \lim\limits_{x\to 0^+} \dfrac{x^3-0}{x-0} = \lim\limits_{x\to 0^+} x^2 = 0$

$\lim\limits_{x\to 0^-} \dfrac{f(x)-f(0)}{x-0} = \lim\limits_{x\to 0^-} \dfrac{x^2-0}{x-0} = \lim\limits_{x\to 0^-} x = 0$

so $f'(0) = \lim\limits_{x\to 0} \dfrac{f(x)-f(0)}{x-0}$ exists and equals 0. $\boxed{\text{So yes, diff. at } x=0.}$

(8)

(a) $f'(x) = \dfrac{x \cdot \sec^2 x - \tan x}{x^2}$

(b) $f'(x) = 2(\sec(3x^5))^1 \cdot \sec(3x^5)\tan(3x^5) \cdot 15x^4$

(c) $f'(x) = e^{\cos x} \cdot (-\csc^2\sqrt{x}) \cdot \dfrac{1}{2\sqrt{x}} + e^{\cos x}(-\sin x) \cdot \cot(\sqrt{x})$

(9) Sps f is a continuous function on [a,b] and

(a) L is any number between f(a) and f(b). Then there exists at least one number c between a and b with $f(c) = L$.

(b) $f(x) = \cos x - 8\tan x$ is continuous on $[0, \pi/4]$

$f(0) = \cos 0 - 8\tan 0 = 1 - 8(0) = 1 > 1/2$

$f(\pi/4) = \cos(\pi/4) - 8\tan(\pi/4) = \frac{\sqrt{2}}{2} - 8 \cdot 1 < 0 < \frac{1}{2}$

Using $L = 1/2$ in the I.V.T we have

$f(0) < L < f(\pi/4)$ so there is a number c in $(0, \pi/4)$ with $f(c) = 1/2$ as needed.