Tufts University

Math 11 **Department of Mathematics** **Spring 2009**

Exam 1 – Solutions

1. (a) $\log_9 3^{-1} = \log_9 9^{-1/2} = -\dfrac{1}{2}$

   (b) $\sin -\dfrac{2\pi}{3} = -\dfrac{\sqrt{3}}{2}$

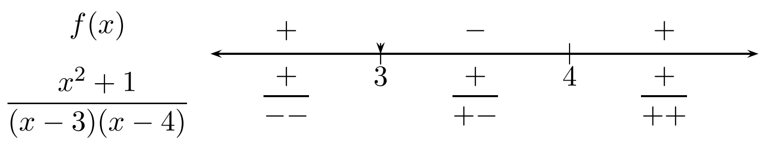
   (c) $\log_{15}(1/2) + \log_{15}(30) = \log_{15}(1/2 \cdot 30) = \log_{15}(15) = 1$

2. (a) $\pm\dfrac{\pi}{6}$. In the first quadrant of the unit circle, $\cos\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$ and so $\cos -\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$ also.

   (b) Raising 2 the power of each side gives $\log_2(x) + 2 = 4$. Subtracting 2 from both side and again raising 2 the power of each side gives $x = 2^2 = 4$.

   (c) $e^{\ln(x)/2} = e^{\ln(x^{1/2})} = \sqrt{x} = 5$ so $x = 25$.

3. (a) If we plug in $x = 3$ we get a non-zero number over zero so the limit is either ∞ or $-\infty$. We have the following sign chart that tells us that limit is $\lim\limits_{x\to 3^+} f(x) = -\infty$.



   (b) $f(x)$ is continuous at $x = 1$ so we can just plug in 1 for x and get $\lim\limits_{x\to 1} f(x) = \dfrac{1^2+1}{(1-3)(1-4)} = \dfrac{1}{3}$.

4. (a) $\lim\limits_{x\to\infty} \dfrac{\sqrt{1+2x+9x^2}}{1+2x} \cdot \dfrac{1/x}{1/x} = \lim\limits_{x\to\infty} \dfrac{\sqrt{1+2x+9x^2}}{1+2x} \cdot \dfrac{1/\sqrt{x^2}}{1/x}$, since $x > 0$.

   So, $= \lim\limits_{x\to\infty} \dfrac{\sqrt{1/x^2+2/x+9}}{1/x+2} = \dfrac{\sqrt{0+0+9}}{0+2} = \dfrac{3}{2}$

   $\lim\limits_{x\to-\infty} \dfrac{\sqrt{1+2x+9x^2}}{1+2x} \cdot \dfrac{1/x}{1/x} = \lim\limits_{x\to-\infty} \dfrac{\sqrt{1+2x+9x^2}}{1+2x} \cdot -\dfrac{1/\sqrt{x^2}}{1/x}$, since $x < 0$.

   So, $= \lim\limits_{x\to-\infty} \dfrac{-\sqrt{1/x^2+2/x+9}}{1/x+2} = \dfrac{-\sqrt{0+0+9}}{0+2} = -\dfrac{3}{2}$

   Thus, $f(x)$ has horizontal asymptotes of $y = 3/2$ and $y = -3/2$.

   (b) $\lim\limits_{x\to 1} \dfrac{\sin(2x-2)}{\sin(3x-3)} = \lim\limits_{x\to 1} \dfrac{\frac{\sin(2x-2)}{x-1}}{\frac{\sin(3x-3)}{x-1}} = \lim\limits_{x\to 1} \dfrac{2\frac{\sin(2x-2)}{2x-2}}{3\frac{\sin(3x-3)}{3x-3}} = \dfrac{2}{3}\dfrac{\lim_{x\to 1}\frac{\sin(2x-2)}{2x-2}}{\lim_{x\to 1}\frac{\sin(3x-3)}{3x-3}}$.

   Now let $u = 2x-2$ and $v = 3x-3$ and note that as $x \to 1$ we have $u \to 0$ and $v \to 0$.

   Thus, above $= \dfrac{2}{3}\dfrac{\lim_{u\to 0}\frac{\sin u}{u}}{\lim_{v\to 0}\frac{\sin v}{v}} = \dfrac{2}{3} \cdot \dfrac{1}{1} = \dfrac{2}{3}$.

5. (a) By the product rule: $y' = (x)' \cdot e^{2x} + x \cdot (e^{2x})' = e^{2x} + 2xe^{2x}$.
   (b) We can rewrite $y = \sin(x+1)^{1/2}$. By the chain rule:
   $y' = \cos(x+1)^{1/2} \cdot \frac{1}{2}(x+1)^{-1/2} = \cos\sqrt{x+1} \cdot \frac{1}{2\sqrt{x+1}}$.
   (c) By the quotient rule $y' = \frac{(1+x^3)' \cdot \cos x - (1+x^3) \cdot (\cos x)'}{\cos^2 x} = \frac{3x^2 \cos x + (1+x^3)\sin x}{\cos^2 x}$

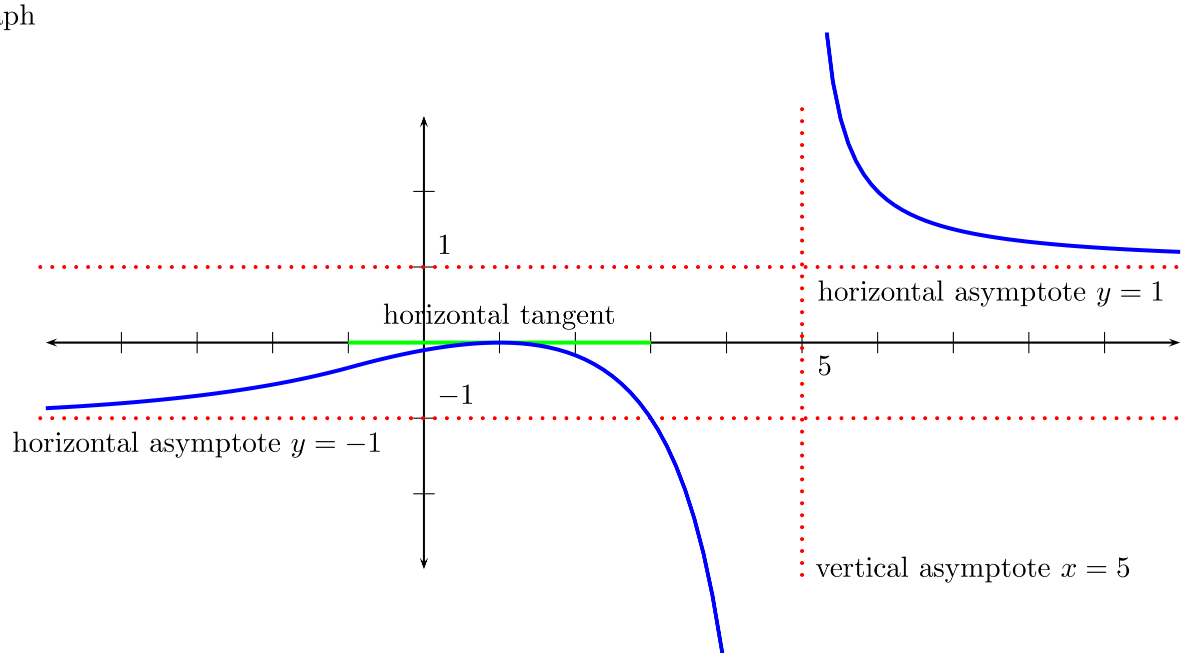
6. (a) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
   (b) $f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \lim_{h \to 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$
   $= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$

7. (a) $y' = F'(x^2) \cdot 2x$. So, $y'(1) = F'(1) \cdot 2 = 2$.
   (b) $y' = 2xF(x) + x^2F'(x)$. So, $y'(1) = 2F(1) + 1F'(1) = 7$.

8. (c) is the correct answer.

9. (a) False: $g^{-1}(1) \approx 1.75$.
   (b) True: $g^{-1}(-1) = 0$.
   (c) True: $g^{-1}(2.75) = 2.75$. (The intersection of the two lines.)

10. graph



11. (a) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^2 + 2x + 2 = 5$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 3x + 2 = 5$

So, $\lim_{x\to 1} f(x) = 5 = f(1)$, thus f is continuous at $x = 1$.

(b) Since the function is continuous, we can look at the derivatives of the parts on either side of 1. We see that $(3x+2)'$ at $x = 1$ is 3 and $(x^2+2x+2)'$ at $x = 1$ is 4, i.e., the slopes from either side are different and the graph has a cusp at $x = 1$.

12. The equation of the line can be written as $y = -\frac{1}{4}x + \frac{1}{4}$ so the slope is $-\frac{1}{4}$.
This gives $y' = \dfrac{-2}{(1+x)^3} = -\dfrac{1}{4}$
cross multiplying gives $(1+x)^3 = 8$
Taking cubed roots gives $1 + x = 2$ so $x = 1$. Plugging into the original function we get the point $(1, 1/4)$ and slope $-1/4$ so the equation of the line is $y - 1/4 = -1/4(x - 1)$.