

Math 32 FINAL Tufts 05/08/2017

YOUR NAME

YOUR SECTION

Problem	Score	Possible Total
1		
2		
3		
4		
5		
6		
7		
TOTAL		(110)

You may not use books, notes, or **calculators** during the exam. Solutions must be written in this exam. Cross out any work you do not want graded. You are required to **sign** the bottom of this exam. With your signature, you pledge that you have neither given nor received assistance on this exam. You must show your work on every problem. Enjoy taking your final!

1. Find the limits.

(a) (6 points)

$$\lim_{x \rightarrow \infty} \frac{\ln(5x^2 + 3)}{\ln(7x^2 + 17)}$$

(b) (6 points)

$$\lim_{x \rightarrow 0} \csc(6x) \sin(7x)$$

2. Short answer. Explain your answer.

(a) (5 points) Give an example of a function $f : [0, 1] \rightarrow [0, 1]$ which does not satisfy the Intermediate Value theorem.

(b) (5 points) Give an example of a continuous function $f : [0, 1] \rightarrow [0, 1]$ which does not satisfy Rolle's theorem.

3. Find the derivatives.

(a) (5 points)

$$f(x) = \int_a^x e^{\sin t} dt$$

(b) (6 points)

$$f(x) = \int_0^{x^3} \sin^3 t dt$$

(c) (5 points)

$$f(\theta) = \tan(\sin(\theta))$$

(d) (6 points)

$$f(x) = x^{\sin x}$$

4. Compute the integrals.

(a) (7 points)

$$\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

(b) (3 points)

$$\int_{-4}^4 (x + x^3 + x^5 + x^7) dx$$

(c) (7 points)

$$\int_0^{\pi/8} \sec^2(2x) dx$$

(d) (7 points)

$$\int \frac{x}{\sqrt{x-6}} dx$$

5. Computing the area between curves. (12 points)

- (a) Graph the functions $f(x) = x^2 - 4x$ and $g(x) = 2x - 8$ on the same xy plane. Shade the region A which is bounded by the graphs of $y = f(x)$, $y = g(x)$ and $y = 0$ and which contains the point $(0, 0)$.
- (b) Set up, but do not evaluate, a sum of integrals giving the area of A (with respect to x).
- (c) Set up, but do not evaluate, a single integral giving the area of A (with respect to y).

6. (15 points) A rectangular flower garden with an area of $30m^2$ is surrounded by a grass border 1 m wide on two sides and 2 meters wide on the other two sides. What dimensions minimize the combined area of the garden and border? Please show all your work. See the attached picture.

7. (15 points) Sketch the graph of

$$f(x) = \frac{2x}{x^2 - 1}$$

You may use that $f'(x) = \frac{-2(x^2+1)}{(x^2-1)^2}$ and $f''(x) = \frac{4x(x^2+3)}{(x^2-1)^3}$. Show and label all (vertical and horizontal) asymptotes, relative max and min, x and y intercepts, and inflection points.