Math 11 | Tufts University
Department of Mathematics
Final Exam | May 9, 2011
8:30 – 10:30 AM

You may not use books, notes, **or calculators** during the exam. Solutions must be written in your exam book; in all problems, you must show all your work in order to receive full credit. Cross out any work you do not want graded. You are required to **sign your exam book**; with your signature, you pledge that you have neither given nor received assistance on this exam.

---

1. (10 points) Evaluate the following indefinite integrals.

(a) $\int \frac{\cos(2x)}{\sin(2x)} dx.$ (b) $\int \frac{x^2}{\sqrt{x+1}} dx$

2. (10 points) An experiment observes a particle moving in a straight line. The position function $s(t)$ represents the distance in meters of the particle from the origin after t seconds. The acceleration of the particle after t seconds is given by the function $a(t) = 2 + e^{t-1}$ meters per second^2. If the velocity of the particle after 1 second is given by $v(1) = 2$ meters per second, and if the position of the particle after 1 second is given by $s(1) = 0.2$ meters, find the position function $s(t)$ of the particle.

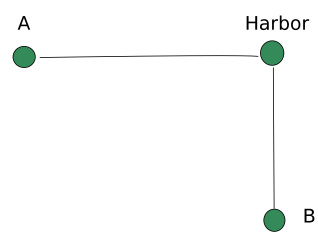
3. (10 points) Evaluate the following definite integrals.

(a) $\int_e^{e^3} \frac{1+2x}{2x} dx$ (b) $\int_{-1}^{1} t^2 \cos(\pi t^3) dt$

4. (10 points) Evaluate the following limits, if they exist. Justify your answers.

(a) $\lim_{x\to\infty} \frac{x+1}{\sqrt{x^4+x+1}}$ (b) $\lim_{x\to\infty} \frac{\ln(x)}{\ln(2x+1)}.$

5. (10 points) At noon, the sailing ship Ahoy There! is 15 miles west of a certain harbor and is sailing east toward the harbor at a rate of 5 miles per hour; also at noon, the ship Bermuda is 6 miles south of the harbor and is sailing south away from the harbor at a rate of 4 miles per hour. Find the rate at which the distance between the two ships is changing at 1:00 pm.



6. (12 points) Find $\frac{dy}{dx}$ for the following expressions. Don't simplify your answers.

(a) $y = \frac{\ln(x+1)}{x+2}$ (b) $y = \sqrt{e^{\sin(x)}+1}$ (c) $y = \int_0^{\ln(x)} t^2 dt$ (d) $y = (3x+1)^x.$

EXAM CONTINUES ON REVERSE SIDE.

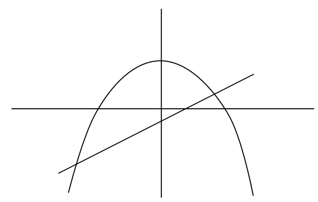
7. (8 points) Consider the curve defined by the equation $1 + xy = x^2 + y^2$.

(a) Use implicit differentiation to find $\frac{dy}{dx}$.

(b) Find the equation of the tangent line to the curve when $(x, y) = (1, 1)$.

8. (10 points)

(a) Find a definite integral representing the area of the planar region bounded by the curves $y = -x^2 + 2$ and $y = 2x - 1$. Don't evaluate your integral.



(b) Evaluate the definite integral $\int_1^4 \sqrt{9 - (x-1)^2}dx$ by interpreting it as an area.

9. (10 points)

(a) Find the absolute minimum value of the function $f(x) = x^3 - 3x + 3$ on the interval $[0, 2]$.

(b) If $h(x) = e^x - x^2$, on what interval(s) is the graph of the function $h(x)$ concave up?

10. (10 points) The base of a cylindrical can of height h cm is to be a circle of radius r cm. If the can is to have volume 54π cm^3, what dimensions minimize the surface area of the can? Be sure to explain why your solution *minimizes* the area.

Recall: The *area* of a circle of radius r is given by πr^2, and the *circumference* of a circle of radius r is given by $2\pi r$. The *volume* of the cylinder is the product of its height and the area of its base. The *surface area* of the cylinder is the sum of the area of its base, the area of its lid, and the area of the side. Finally, the area of the side of the cylinder is the product of its height and the circumference of its base.

END OF EXAM.