Tufts University

Math 11 | **Department of Mathematics** | **December 15, 2011**

Final Exam | **8:30 to 10:30 a.m.**

Instructions: No calculators, notes or books are allowed. You should show all work to receive full credit. Please circle your answers and cross out any work you do not want graded. Remember to sign your blue book, indicating that you have neither given nor received assistance on this exam.

Please start each question on a fresh page in your blue book.

1. (10 pts) True/False Questions. No justification required. (No partial credit.)

   (a) The function $f(x) = \dfrac{x^2 - x}{x^2 - 1}$ has a vertical asymptote at $x = -1$ and at $x = 1$.

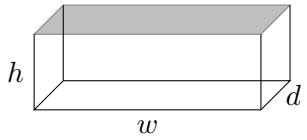
   (b) The domain of $f(x) = \ln(x^2 + 1)$ is $(0, \infty)$.

   (c) $\lim\limits_{x \to \infty} \dfrac{\ln x}{e^x} = \infty$.

   (d) $\displaystyle\int_{-2}^{0} \sqrt{4 - x^2}\, dx = \pi$.

   (e) $\displaystyle\int x^2 e^x\, dx = \frac{x^3}{3} \cdot e^x + C$.

2. (12 pts) A rectangular box has three dimensions: height h, depth d, and width w. The postal service puts a fancy label on top of the box; they will accept a box for shipment only if the sum of the height of the box and the perimeter of the top does not exceed 54 inches. Suppose that you want to ship a box with a square side, as in the picture, with the largest possible volume. What are the dimensions of that box? Show your work.



3. (10 pts) Consider the function $f(x) = \dfrac{1}{x}$.

   (a) Use the limit definition of the derivative to compute the slope of the tangent line to $f(x)$ at $x = 2$.

   (b) Using any method, find the equation of the tangent line to $f(x)$ at $x = 1$.

4. (12 pts) Compute the first derivative of each of the following functions. Do not simplify.

   (a) $F(x) = \displaystyle\int_0^{x^3} 4u^2 + 2\, du$

   (b) $f(x) = \arctan(\ln(x^2 - 4x))$

5. (12 pts) Compute each of the following limits using any method. JUSTIFY YOUR WORK.

(a) $\lim_{x\to\infty} \dfrac{3x+1}{\sqrt{x^3+2x+5}}$ (b) $\lim_{x\to 1}\left(\dfrac{x}{x-1}-\dfrac{1}{\ln x}\right)$

6. (10 pts) Consider the function $f(x)=\dfrac{x^2+x+1}{x^2}$, which has first derivative $f'(x)=\dfrac{-x-2}{x^3}$ and second derivative $f''(x)=\dfrac{2x+6}{x^4}$. NO JUSTIFICATION REQUIRED.

(a) Find any horizontal or vertical asymptotes of f.

(b) For what values of x is f increasing?

(c) Give the x–coordinates of all local maxima and local minima of f. Which are max and which are min?

(d) On what intervals is f concave up?

(e) Give the x–coordinates of all inflection points of f, if there are any.

7. (10 pts) Consider the integral $\displaystyle\int_0^{2\pi} \sin x\ dx$.

(a) Graph $y=\sin x$ on the interval $[0,2\pi]$ and draw the picture for the midpoint Riemann sum for the regular partition with $n=3$. Evaluate the Riemann sum as a number.

(b) Using your graph of $y=\sin x$, give the value of the integral exactly just from the picture.

8. (12 pts) Consider the area function $A(x)=\int_2^x \frac{1}{2}t^2\ dt$.

(a) Draw a diagram to show what area this is computing.

(b) Compute $A(4)$ exactly. (Your answer should be a number.)

(c) Evaluate the definite integral to give a formula for $A(x)$.

(d) Using your formula for $A(x)$ from the last part, find $A'(x)$. (Your answer should only have x in it, and not t.)

One version of the Fundamental Theorem of Calculus says that $\dfrac{d}{dx}\displaystyle\int_a^x f(t)\ dt = f(x)$. Does this match with your answer from the previous part?

9. (12 pts) Compute the following integrals.

(a) $\displaystyle\int \frac{m^3-m+1}{m^2}\ dm$ (b) $\displaystyle\int 3\sin x\cos x\ dx$ (c) $\displaystyle\int_{1/2}^{3} \frac{1}{\sqrt{3+2x}}\ dx$

End of Exam