Tufts University
Department of Mathematics
Math 11, Final Exam

Thursday, December 11, 2008 **8:30–10:30 a.m.**

No books, notes or electronic devices of any kind are allowed. Please do all of your work in the blue book. Remember to sign your exam book. With your signature, you are pledging that you have neither given nor received assistance on the exam.

1. [12 points] Find y'.

(a) $y = \dfrac{\ln x}{x^3 + 2}$ (b) $y = \tan^{-1}(e^x)$ (c) $y = x^{1/x}$

(d) $xy^2 = e^y$ (You may express y' in terms of x and y.)

2. [9 points] Find the limits.

(a) $\displaystyle\lim_{x\to 1} \frac{x^2 - 1}{x^2 - 3x + 2}$ (b) $\displaystyle\lim_{x\to 0+} x^2 \ln x$ (c) $\displaystyle\lim_{x\to 0} \frac{x - \sin x}{x - \tan x}$

3. [14 points] Consider the function

$$f(x) = \frac{x-1}{x^2}.$$

The first two derivatives of $f(x)$ are

$$f'(x) = \frac{2-x}{x^3} \qquad \text{and} \qquad f''(x) = \frac{2(x-3)}{x^4}.$$

Answer the following questions about $f(x)$. Note that "none" is a possible response.

(a) What are the equations of any horizontal asymptotes?

(b) What are the equations of any vertical asymptotes?

(c) On which open intervals is $f(x)$ increasing?

(d) What are the x-coordinates of the points where $f(x)$ has a local maximum?

(e) On which open intervals is $f(x)$ concave up?

(f) What are the x-coordinates of all inflection points?

(g) Sketch the graph of $f(x)$.

4. [7 points] (a) Let $s(t)$, $v(t)$, and $a(t)$ be the position, velocity, and acceleration at time t of a particle moving on a line. Find $s(t)$ assuming that

$$a(t) = \cos t - e^t - 1, \quad s(0) = 0, \quad v(0) = 5.$$

(b) Suppose that the position $f(t)$ of another particle moving on a line satisfies $f(1) = -3$ and $f(10) = 6$ and that f is differentiable for all values of $t > 0$. What does the Mean Value Theorem tell you about the velocity $f'(t)$ of this particle?

5. [12 points] (a) Two ships start at the same dock. At noon, the Titanic begins traveling east at a constant rate of 5 mph, and the Black Pearl begins traveling north at a constant rate of 12 mph. How fast is the distance between the ships changing at 1:00 pm?

(b) A rectangular box with square base and without a top is to be made from 48 ft^2 of material. What dimensions will result in a box with the largest possible volume? Say why your answer is a maximum.

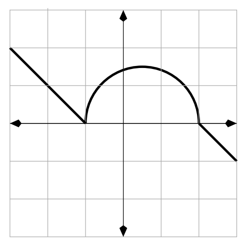
6. [9 points] (a) Find $\sin^{-1}(\frac{1}{2})$.

(b) Find the equation of the tangent line to the curve $f(x) = e^x + \sin^{-1}(x)$ at the point $(0, 1)$.

(c) Find the domain of the function $g(x) = \sin^{-1}(2x - 3)$.

7. [8 points] (a) Evaluate the Riemann sum for $f(x) = x^3$, where $2 \leq x \leq 11$, with 3 subintervals, taking sample points to be the right endpoints. Do not simplify your answer.

(b) Evaluate $\int_{-3}^{3} g(x)\, dx$, where $g(x)$ is the function whose graph is as shown:



8. [9 points] (a) Find the derivative of the function $f(x) = \int_2^x \frac{t}{1+t^3}\, dt$.

(b) Find the derivative of the function $g(x) = \int_3^{x^2} e^{\sin t}\, dt$.

(c) Find the x-value of the local maximum of the function $h(x) = \int_0^x \frac{(t-1)(t-2)}{t-5}\, dt$ on the interval $(0, 3)$.

9. [12 points] Evaluate the integrals.

(a) $\int_1^2 \frac{e^{1/x}}{x^2}\, dx$ (b) $\int x \sin(x^2)\, dx$

(c) $\int \frac{1+x}{1+x^2}\, dx$ (d) $\int_{-1}^{1} x\sqrt{x+1}\, dx$

10. [8 points] Set up (but do not evaluate) the integrals for the following quantities.

(a) The area bounded by the graphs of $y = e^x$, $y = 2e^{-x}$ and $x = 0$.

(b) The volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = 4$ about the x-axis.

END OF EXAM