Tufts University

Math 11 | Department of Mathematics | April 11, 2011

Exam 2

You may not use books, notes, **or calculators** during the exam. Solutions must be written in your exam book; cross out any work you do not want graded. You are required to **sign your exam book**; with your signature, you pledge that you have neither given nor received assistance on this exam.

1. (6 points) Indicate the correct value for each of the following expressions in your exam book.

(a) $\cos(\arcsin(-3/5))$

(i) $-3/5$ (ii) $-4/5$ (iii) $4/5$ (iv) 4

(b) $\arctan(\tan(5\pi/4))$

(i) $\pi/4$ (ii) $-\pi/4$ (iii) $5\pi/4$ (iv) $-5\pi/4$

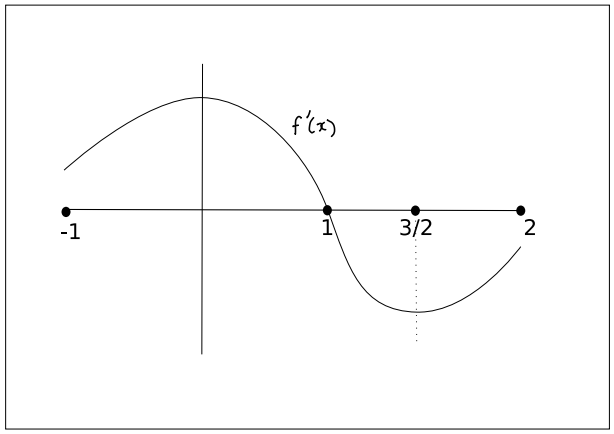
2. (15 points) Find the derivatives. Don't simplify your answers.

(a) $y = \arctan(\sqrt{x+1})$ (b) $y = \cos(x)^{2x}$ (c) $y = \dfrac{\ln(x)}{x^2}$

3. (12 points) A spotlight on the ground shines on a wall 20 m distant. A boy 1 m tall walks from the wall towards the light at a rate of 1 m/sec. At what rate is the length of the boy's shadow on the wall changing when the boy is 10 m from the searchlight?

4. (8 points) Find the equation of the tangent line to the curve defined by the equation $1 + \sqrt{xy} = 1 + x^2y$ at the point $(x, y) = (1, 1)$.

5. (12 points) The following represents the graph of the *derivative* $f'(x)$ of the function $f(x)$ on the interval $(-1, 2)$.



(a) Find the critical numbers of f in the interval $(-1, 2)$. For each critical number, indicate whether f has a local maximum, a local minimum, or neither.

(b) On which open interval(s) in the interval $(-1, 2)$ is the graph of f concave up?

(c) At which value(s) of x in the interval $(-1, 2)$ does f have an inflection point?

EXAM CONTINUES ON REVERSE

6. (7 points) Compute $\lim_{x \to 0} \dfrac{e^{2x} - 1 - 2x}{x^2}$.

7. (12 points) For the function $f(x) = x^3 - 3x + 1$ on the interval $[-3, 0]$, find:

(i) all critical numbers of $f(x)$ in the interval $(-3, 0)$.

(ii) the absolute maximum *value* of $f(x)$ on the interval $[-3, 0]$.

(iii) the absolute minimum *value* of $f(x)$ on the interval $[-3, 0]$.

8. (12 points) A box with no lid and a square base is to have a volume 125 cm^3. If the cost of the material for the sides is half of the cost of the material for the base, what dimensions will minimize the costs of the material used to make the box?

9. (16 points) Let $f(x) = \dfrac{2x^2}{x^2 - 1}$. Then

$$f'(x) = \frac{-4x}{(x^2-1)^2} \qquad \text{and} \qquad f''(x) = \frac{12x^2 + 4}{(x^2-1)^3}$$

(a) What is the domain of $f(x)$?

(b) Find the interval(s) of increase of $f(x)$.

(c) Find the interval(s) of decrease of $f(x)$.

(d) At which x value(s) does $f(x)$ have a local maximum?

(e) At which x value(s) does $f(x)$ have a local minimum?

(f) Find the interval(s) on which $f(x)$ is concave up.

(g) Find the interval(s) on which $f(x)$ is concave down.

(h) Are there any inflection points? If so, at which x value(s) do they occur?

END OF EXAM.