Tufts University

Math 32 | **Department of Mathematics** | **November 19, 2012**

Exam 2 | **12:00 to 1:20 p.m.**

Instructions: No calculators, notes or books are allowed. You should show all work to receive full credit. Please circle your answers and cross out any work you do not want graded. Remember to sign your blue book, indicating that you have neither given nor received assistance on this exam.

Part I. (No Partial Credit) Write in a column on the inside cover of your bluebook, the labels for the various parts of questions 1 and 2: 1a) through 1e) and 2A) through 2J). Put your ANSWERS ONLY to questions 1 and 2 next to the appropriate part. Your work can be done on your exam sheet or elsewhere in the bluebook.

1. (10 pts) Answer each of the following True/False questions.

   (a) If $f(x)$ is a continuous function on the interval $[0,1]$ with $f(0) = 0 = f(1)$, then there must be a point $c \in (0,1)$ with $f'(c) = 0$.

   (b) Suppose c is an interior point of the domain of the function $f(x)$ and $f'(c) = 0$, then $f(x)$ must have either a local maximum or a local minimum at $x = c$.

   (c) Suppose the continuous function $f(x)$ satisfies: $f''(x) > 0$ for $x > 2$, $f''(x) < 0$ for $x < 2$. Then function $y = f(x)$ has an inflection point at $(2, f(2))$.

   (d) The graph of $y = x^{\sqrt{x}}$ has no horizontal tangent lines.

   (e) The linear approximation for $\sqrt{24.98} = 5 - \frac{1}{10}(.02)$.

2. (20 pts) Consider the following function with it's first and second derivatives in simplified form:

$$f(x) = \frac{\ln x}{x^2} \quad f'(x) = \frac{1 - 2\ln x}{x^3} \quad f''(x) = \frac{6\ln x - 5}{x^4}$$

   "None" could be an appropriate answer for some of the statements below.

   (A) The **domain** of $f(x)$ is

   (B) A **y-intercept** of the graph of $y = f(x)$ occurs at $y =$

   (C) An **x-intercept** of the graph of $y = f(x)$ occurs at $x =$

   (D) The graph of $y = f(x)$ has a **horizontal asymptote** at $y =$

   (E) The graph of $y = f(x)$ has a **vertical asymptote** at $x =$

   (F) The function $f(x)$ is **increasing** on the open interval(s)

   (G) The function $f(x)$ has a **local minimum** at $x =$

   (H) The function $f(x)$ has a **local maximum** at $x =$

   (I) The function $f(x)$ is **concave up** on the open interval(s).

   (J) The graph of $y = f(x)$ has an **inflection point** at $x =$.

EXAM CONTINUES ON THE OTHER SIDE

Part II. Show all of your work to receive full credit on these.

3. (18 pts) A) Sketch and label the graph of **ONE** function $y = f(x)$ which satisfies all the conditions (a)-(h) below:

   (a) $f(0) = 4$, $f(1) = -1$, $f(2) = 0$, f is differentiable at all x except $x = 0$ and $x = 1$;

   (b) $\lim_{x\to 0-} f(x) = -4, \quad \lim_{x\to 0+} f(x) = 4, \quad \lim_{x\to 1} f(x) = -1$

   (c) $\lim_{x\to -\infty} f(x) = 0$

   (d) $\lim_{x\to \infty} f(x) = -4$

   (e) $f'(x) > 0$ for x in $(1, 2)$;

   (f) $f'(x) < 0$ for $x < 0, \quad 0 < x < 1, \quad x > 2$;

   (g) $f''(x) > 0$ for $0 < x < 1$ and $x > 3$;

   (h) $f''(x) < 0$ for x in $(-\infty, 0)$ and $(1, 3)$;

   B) State the Extreme Value Theorem.

   C) What conclusion(s) of the extreme value theorem fail for the above function on the interval $[-1, 1]$? Additionally, specify the assumption(s) of the extreme value theorem which are not satisfied by the above function.

4. (12 pts)

   (a) Find the slope of the curve $y = \ln(e^{3x^2} \cdot \tan^{-1} x)$ when $x = 1$.

   (b) Find the slope of the curve $x^3 + y^3 = 2xy$ at the point $(1, 1)$.

5. (10 pts) Consider the function $f(x) = 2x^{4/3} - x^{1/3}$.

   (a) Find all critical points of $f(x)$.

   (b) Find the absolute maximum and absolute minimum values of $f(x)$ on the interval $[0, 1]$.

6. (6 pts) Suppose f is differentiable and continuous everywhere and has two real roots. Show that $f'(x)$ must have at least one real root using the Mean Value Theorem.

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7. (12 pts) Calculate the following limits. Specify above each equality for which l'Hôpital's Rule is used the appropriate version of $0/0$ or ∞/∞.

   (a) $\displaystyle\lim_{x\to\infty} \frac{x^2 + e^x}{x + e^x}$

   (b) $\displaystyle\lim_{x\to 0+} (1 + x)^{3\cot x}$

8. (12 pts) Find the point on the curve $y = \sqrt{x}$ whose distance from the point (4,0) is minimal. Draw a picture and be sure to give appropriate justification for your conclusion.

End of Exam