Tufts University Department of Mathematics Exam II

No books, notes, or calculators are allowed on the exam. SHOW ALL WORK. Remember to sign your exam book. With your signature, you are pledging that you have neither given nor received help in this exam.

1. (12 points) Evaluate in parts (a) and (b). Simplify in part (c).

(a)
$$\sin^{-1}(\sin\frac{7\pi}{6})$$
 (b) $\tan^{-1}(-\sqrt{3})$ (c) $\tan(\cos^{-1}(x))$

2. (20 points) Find the derivative. You do \underline{not} need to simplify your answer.

(a)
$$y = \frac{\ln x}{\sin^{-1} x}$$

(b) $f(t) = \tan^{-1}(e^{3t})$
(c) $g(x) = \log_3(\cos x)$
(d) $h(x) = (\frac{1}{x})^{\ln x}$

3. (14 points) For each of the following functions find the x and y coordinates of an absolute minimum and an absolute maximum in the indicated interval. Include in your answer a list of all critical points and end points.

(a)
$$f(x) = -x^{4/5}$$
 on $[-1, 32]$.

- (b) $g(x) = \frac{x}{x^2 + 1}$ on [0, 3].
- 4. (10 points) Find the equation of the tangent line to the curve

$$2x^3y + x^2 = 4e^y$$

through the point (2,0).

- 5. (8 points)
 - (a) Let $f(x) = x^3 + 2x 3$. According to the Mean Value Theorem, there is a point c in the open interval (-1, 2) such that f'(c) =____.
 - (b) Draw the graph of a function whose domain is the closed interval [1, 3] but which does <u>not</u> have an absolute maximum in this interval.
- 6. (10 points) Let $f(x) = x + 2 \cos x$. Where in the interval $[0, \pi]$ is f increasing and where is it decreasing? Where in this interval is f convex up and where is it convex down?

Exam continues on other side

Math 11

- 7. (12 points) On a cloudless fall afternoon a flying saucer is descending vertically and noiselessly at a rate of 20 feet/min. to a point P in the middle of the Tufts soccer field. A small turtle is rooted in fear at a spot 40 feet from the point P. As what rate is the distance between the flying saucer and the turtle changing when the flying saucer is 30 feet above the ground?
- 8. (14 points) Given

$$f(x) = 3x^5 - 20x^3 + 7 \qquad f'(x) = 15x^4 - 60x^2 \qquad f''(x) = 60x^3 - 120x$$

- (a) Find the interval(s) on which the graph of f(x) is increasing.
- (b) Find the interval(s) on which the graph of f(x) is decreasing.
- (c) Find the x coordinate of all local maxima.
- (d) Find the x coordinate of all local minima.
- (e) Find the interval(s) on which the graph of f(x) is concave up.
- (f) Find the interval(s) on which the graph of f(x) is concave down.
- (g) Find the x coordinate of all inflection points.

Note: The answer to any of the above may be none.

End of exam.