Math 32 | **TUFTS UNIVERSITY** | **February 29, 2016**
Calculus I | **Department of Mathematics** | **12-1:20 pm**
All sections | **Exam I**

No books, notes, **or calculators**. **TURN OFF YOUR CELL PHONE. ANYONE CAUGHT WITH THEIR CELL PHONE ON WILL BE GIVEN A 10 POINT DEDUCTION.** Cross out what you do not want us to grade. You **must** show work to receive full credit. Please try to write neatly. You need not simplify your answers unless asked to do so. You should evaluate standard trigonometric functions like $\tan(\pi/3)$. You are required to **sign** your exam book. With your signature, you pledge that you have neither given nor received assistance on this exam.

Problem	Point Value	Points
1	10	
2	6	
3	9	
4	12	
5	18	
6	12	
7	6	
8	6	
9	10	
10	6	
11	5	
	100	

1. (10 points) The Derivative

   (a) State the definition of the derivative of a function $y = f(x)$ as a limit.

   (b) Use your definition to find the derivative of the function $f(x) = \dfrac{1}{5x + 2}$.

2. (6 points) Find the following limits or state that they do not exist. If you are able to describe a nonexistent limit by using the notation ∞ or $-\infty$ please do so. Make sure you show your work.

(a) $\lim_{x \to -3^-} \dfrac{2x}{(x+3)(x^2-9)}$

(b) $\lim_{x \to 0} \dfrac{\sin(4x)}{2x}$

3. (9 points) Find the following limits or state that they do not exist. If you are able to describe a nonexistent limit by using the notation ∞ or $-\infty$ please do so. Show your work. (In some cases a graph will suffice.)

(a) $\lim_{x\to\infty} e^{-2x}$

(b) $\lim_{x\to-\infty} \dfrac{14x^2+12}{10-x-x^2}$

(c) Use the Squeeze Theorem to find $\lim_{x\to\infty} \dfrac{\sin x+3}{x^2}$

4. (12 points) Differentiate. DO NOT SIMPLIFY YOUR ANSWERS.

(a) $y = \sin^2(x)$

(b) $y = 10e^{3x}$

(c) $y = \ln(6 - x)$

(d) $y = 2^x$

5. (18 points) Differentiate. DO NOT SIMPLIFY YOUR ANSWERS.

(a) $y = \cos^6(e^{-2x})$

(b) $y = \dfrac{e^x + x^{2/3}}{\tan x}$

(c) $y = x^{\cos x}$

6. (12 points) Differentiation. DO NOT SIMPLIFY YOUR ANSWERS.

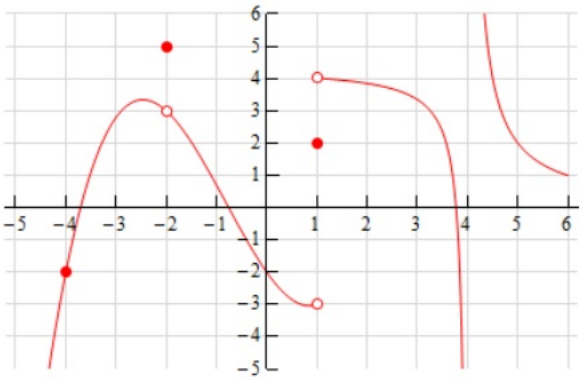
(a) Find the second derivative of the function $y = e^{-5x} \cos x$

(b) Use logarithmic differentiation of find the derivative of $y = \dfrac{(x-2)^8(x+5)^{2/3}}{\sqrt{x-9}}$

7. (6 points) Find an equation for the line tangent to $f(x) = \sin(4x)$ at the point where x $= \frac{\pi}{6}$. You may leave your equation in point-slope form.

8. (6 points) Find the slope of the line tangent to $x^4 - x^2y + y^4 = 1$ at the point $(-1, 1)$.

9. (10 points) Here is the graph of a function $y = f(x)$ drawn in the Cartesian plane.



Complete:

(a) i. $\lim_{x \to -2^-} f(x)$

ii. $\lim_{x \to -2^+} f(x)$

iii. $\lim_{x \to -2} f(x)$

iv. Is $f(x)$ continuous at $x = -2$? Why or why not.

(b) i. $\lim_{x \to 1^-} f(x)$

ii. $\lim_{x \to 1^+} f(x)$

iii. $\lim_{x \to 1} f(x)$

iv. Is $f(x)$ continuous at $x = 1$? Why or why not.

(c) List all intervals on which $f(x)$ is continuous.
(Assume the function is continuous for $x < -4$ and $x \geq 6$.

10. (6 points) Determine whether or not $f(x)$ is continuous at $x = 3$. Show all work.

$$f(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{if } x \neq 3 \text{ is} \\ 6 & \text{if } x = 3 \end{cases}$$

11. (5 points) Let $f(x) = -2x^3 + 6x + 3$. Then $f(x)$ is continuous for all real numbers x. Use the Intermediate Value Theorem to prove that the equation $-2x^3 + 6x + 3 = 0$ has a solution on the interval $(0, 2)$.

Scratch paper

Math 32 Exam I February 29, 2016

Name _____

Circle your section:

32-01 Hao Liang TWF 9:30-10:20

32-02 Mary Glaser TRF 12-12:50

32-03 Garret LaForge TR 1:30-2:20, F 2:30-3:20

32-04 Aliska Gibbins TRF 8:30-9:20

I pledge that I have neither given nor received assistance on this exam.

Signature _____