

**Instructions:** No calculators, notes or books are allowed. You should show all work to receive full credit. Please circle your answers and cross out any work you do not want graded. Remember to sign your blue book, indicating that you have neither given nor received assistance on this exam.

1. (8pts) True/False Questions. You do not need to justify your answers. There will be no partial credit.
  - (a) If the function  $f$  is continuous at  $x = a$ , then  $f$  is also differentiable at  $x = a$ .
  - (b)  $f(x) = \frac{x^2 - 7x + 12}{x - 3}$  has a vertical asymptote at  $x = 3$ .
  - (c) If  $C(x)$  is the cost in dollars of producing  $x$  DVD players, then  $C'(100)$  is the average cost of producing the first 100 DVD's.
  - (d) It is possible for  $\lim_{h \rightarrow 0^+} \frac{f(3+h) - f(3)}{h} = -1$  and  $\lim_{h \rightarrow 0^-} \frac{f(3+h) - f(3)}{h} = 1$ .
  
2. (10pts) A stone is set in motion from a cliff. Its height in feet after  $t$  seconds is given by:  $s(t) = -16t^2 + 100$ . Be sure to give correct units in the parts below.
  - (a) How high is the cliff?
  - (b) Was the stone dropped or tossed upwards?
  - (c) How long does it take the stone to hit the ground?
  - (d) What is the velocity of the stone after  $t = 3$  seconds?
  - (e) What is the acceleration of the stone after 3 seconds? after 4 seconds?
  
3. (8pts) Sketch the graph of a **single** function  $g$  that has **all** of the following properties.
  - (a) The domain of  $g$  is  $x \geq 0$  with  $x \neq 3$ .
  - (b)  $g(0) = 1$ ,  $g(2) = 5$ .
  - (c)  $\lim_{x \rightarrow 2} g(x) = 3$
  - (d)  $\lim_{x \rightarrow 3^-} g(x) = -\infty$ ,  $\lim_{x \rightarrow 3^+} g(x) = \infty$ .
  - (e)  $\lim_{x \rightarrow \infty} g(x) = 5$ .

**EXAM CONTINUES ON THE OTHER SIDE**

4. (16pts) Compute the following limits. Justify your answers.

(a)  $\lim_{x \rightarrow 0} \frac{3 + \tan x}{1 + \sqrt{x}}$

(b)  $\lim_{x \rightarrow 0} \frac{\sin 3x}{x \cos 4x}$

(c)  $\lim_{x \rightarrow \infty} \frac{2 + 5 \cos x}{\ln x}$  (Hint: Use Squeeze Theorem)

(d)  $\lim_{x \rightarrow \pi/2} \frac{\cos x - \cos(\pi/2)}{x - \pi/2}$

5. (9pts) Find any vertical asymptotes of the function  $f(x) = \frac{x^3 - 2x^2 - 3x}{x^2 + 5x}$ . Justify your answer with the appropriate limit calculations.

6. (9pts) Find any horizontal asymptotes of the function  $f(x) = \frac{x^2 + 3}{x\sqrt{2x^2 + 5x + 3}}$ . Justify your answer with the appropriate limit calculations.

7. (16pts) Consider the function  $f(x) = \sqrt{x + 1}$ .

(a) Use the limit definition of the derivative to compute the derivative  $f'(x)$ .

(b) Find a point on the curve  $f(x) = \sqrt{x + 1}$  at which the tangent line to the curve is parallel to the line  $y = x/6$ .

8. (18pts) Compute the first derivative of each of the following functions. DO NOT SIMPLIFY.

(a)  $f(x) = \frac{\pi + \sec 3x}{x^2 + 8}$

(b)  $y = (\tan(\sqrt{x} + 3x))^5$

(c)  $f(x) = e^{x^3+2} \cos(1 + x^{5/2})$

9. (6pts)

(a) (5 pts) Use the Intermediate Value theorem to show there is at least one point in the interval  $(0, \pi/2)$  where the function  $f(x) = 4x/\pi + \sin x$  takes the value 2.

(b) (1 pt) Give a reason why there should be only one such point in the interval  $(0, \pi/2)$ .

**End of Exam**