Tufts University

Math 32 | Department of Mathematics | Spring 2014

Exam 1 Review

1. Compute the following limits if they exist. Otherwise explain why they don't exist.

   (a) $\lim_{x\to -2} \dfrac{x^2+5x+6}{x+2}$.

   (b) $\lim_{x\to\infty} \dfrac{2x-1}{\sqrt{6x^2+2}}$.

   (c) $\lim_{x\to -2} \dfrac{x^2+7x+10}{2x^2+5x+2}$.

   (d) $\lim_{x\to -1} \dfrac{|x+1|}{x+1}$.

   (e) $\lim_{x\to\infty} \dfrac{3x^2+2x+1}{5x^2-x}$.

   (f) $\lim_{x\to\pi} \ln(\cos^2 x)$.

   (g) $\lim_{x\to\infty} (\sqrt{x^2+4x}-x)$.

   (h) $\lim_{x\to\infty} e^{-x(3+\sin x)}$.

   (i) $\lim_{x\to 0} \dfrac{\sin^2(3x)}{x\cot(4x)}$.

2. Compute the derivative of the following functions. Do not simplify.

   (a) $y = \dfrac{x^2-\sqrt{x}}{3x+2\sqrt{x}}$.

   (b) $y = (4x^3 - 2\cos x + 5e^2)^{1/3}$.

   (c) $y = \sqrt{x}e^{2\tan x}$.

   (d) $y = e^{\sec x + 7x}$.

   (e) $y = \dfrac{1-xe^x}{x+e^x}$

   (f) Find dy/dx at $(0,0)$ when $y = e^y(2+\sin x) - 2$

3. Solve the following equations for x.

   (a) $\ln(4-e^{-x}) = -x$ (b) $\ln(\sqrt{x}+x) + \ln(\sqrt{x}-x) = 0$.

4. Evaluate the following.

   (a) $2\ln 6 + \ln\dfrac{3}{4}$. (b) $\ln\tan(\frac{4\pi}{3}) - \ln\sin(\frac{\pi}{6})$. (c) $\csc(-9\pi/4)$

5.
   (a) State the limit definition of the derivative of a function $f(x)$.

   (b) Use the definition to compute the derivative of $f(x) = \sqrt{x^2+1}$ and of $f(x) = \dfrac{1}{3x-1}$.

6. Find all horizontal and vertical asymptotes of the curve $y = \dfrac{x^2-x-6}{x^3-9x}$. Make sure to compute the appropriate limits.

7. Find the equation of the tangent line to the curve $y = 2xe^x$ at the point $(0,0)$.

8. Find the equation of the tangent line to the curve $y = \dfrac{1}{(1+x)^2}$ which is parallel to the line $4y + x = 1$.

9. Graph the function $f(x)$ on $[-2, 7]$ which has all of the following properties:

   On $[-2, 1)$, $f(x) = -x^2 + 3$ and $f(x)$ is piecewise linear on $[1, 7]$ with
   $f(1) = 2$, $\lim_{x\to 3-} f(x) = 3$, $f(3) = -1, f(5) = 4, f(7) = -1$.

   Answer questions (a)-(e), (i) first using your graph alone and ii) second by writing the exact equations defining $f(x)$ and doing the necessary computations.

   (a) On which intervals is f continuous? For any point at which f is discontinuous, explain why.
   (b) On which intervals is f differentiable? For any point at which f is not differentiable, explain why.
   (c) What is $f'(0)$?
   (d) What is $f'(4)$?
   (e) Graph $f'(x)$ on $[-2, 7]$.

   After completing the previous steps, do the following:

   (f) Graph $f'(x)$. What can you determine about $f(x)$ directly from your graph of $f'(x)$ alone.
   [Note: you will not be able to determine $f(x)$ exactly.]

10. Make sure you can graph from memory the following basic trig functions: $\sin x, \cos x, \tan x, e^x, \ln x$ as well as transformations of each such as: $y = 4\cos(x/3)$ and $y = 1 - \tan(3x)$ (Book practice: problem 29 (a-c, f) p. 48, graph each part yourself, then match it with the appropriate graph, finally check answer in back of the book). Also make sure that you can evaluate trig functions at all possible possible or negative multiples of $\pi, \pi/2, \pi/3, \pi/4, \pi/6$. (Book practice: pp. 36-37: Example 1 and Quick check 2.)