Math 32 | Tufts University
Department of Mathematics
Exam 2 | April 7, 2014
12:00-1:20

Instructions: No phones, calculators, notes or books are allowed. **You must show all work to receive full credit, unless otherwise indicated.** Please circle your answers and cross out any work you do not want graded. Remember to sign your blue book, indicating that you have neither given nor received assistance on this exam.

Part I. Fill in the blank. (No Partial Credit except on 1J.)
On the inside of the front cover of your bluebook, write in a column labels for the various parts of question 1: 1A) through 1J). Record next to each label in your bluebook, your ANSWER ONLY for the accompanying blank below with the exception of 1J which requires justification. Work to obtain your answers in (1A-1I) can be done on your exam sheet or elsewhere in the bluebook.

1. (24 pts) Consider the following function with its first and second derivatives in simplified form:

$$f(x) = x^{1/3}\ln x \qquad f'(x) = \frac{x^{-2/3}}{3}(3+\ln x) \qquad f''(x) = -\frac{1}{9}x^{-5/3}(3+2\ln x).$$

"None" could be an appropriate answer for some of the statements below:

(1A) (2 pts) The domain of $f(x)$ is _____.

(1B) (2 pts) An x-intercept of the graph of $y = f(x)$ occurs at $x =$ _____.

(1C) (2 pts) The graph of $y = f(x)$ has a horizontal asymptote at $y =$ _____.

(1D) (2 pts) The function $f(x)$ is increasing on the open interval(s): _____.

(1E) (2 pts) The function $f(x)$ has a local minimum at $x =$ _____.

(1F) (2 pts) The function $f(x)$ has a local maximum at $x =$ _____.

(1G) (2 pts) The function $f(x)$ is concave down on the open interval(s): _____.

(1H) (2 pts) The function $f(x)$ is concave up on the open interval(s): _____.

(1I) (2 pts) The graph of $y = f(x)$ has an inflection point at $(x, y) =$ _____.

(1J) (6 pts, partial credit available on this part only) Determine whether or not the graph of $f(x)$ has a vertical asymptote at $x = 0$? Justify your answer on this part only.

Part II. Show all of your work to get full credit on these problems.

2. (8 pts) Sketch and label the graph of ONE function $y = f(x)$ defined on $(-\infty, \infty)$ that satisfies all the conditions (a) - (h) below:

(a) $f(-2) = f(0) = 0,\ f(-1) = f(3) = 4,\ f(4) = 3,\ f$ is differentiable except at $x = -1$ and $x = 4$;
(b) $\lim_{x\to\infty} f(x) = 5$;
(c) $\lim_{x\to -1+} f(x) = -\infty$
(d) $f'(x) > 0$ on $(-\infty, -1), (-1, 3), (4, \infty)$, ;
(e) $f'(x) < 0$ on $(3, 4)$;
(f) $f''(x) > 0$ on $(-3, -1)$;
(g) $f''(x) < 0$ on $(-\infty, -3), (-1, 4), (4, \infty)$.

3. (18 points) Find dy/dx and DO NOT SIMPLIFY your answer.

   a) $y = \ln\left(\dfrac{e^x x^{2/3}}{\sqrt{1+x^4}}\right)$, do this part by using properties of logarithms

   b) $y = (2e^x + \sin x)^{\ln x}$, $\quad x > 0$, do this part by using logarithmic differentiation

   c) $y = \sin^{-1}\left(\dfrac{\cos^2 x}{8}\right)$

4. (12 pts) For each of the limits below, first identify which indeterminate form is represented by the limit; "none" is a possible answer. Second, find the limit; each time you apply l'Hôpital's rule to obtain an equality of limits, specify above the equality symbol the notation $0/0$ or ∞/∞ as appropriate.

   (a) $\lim\limits_{x\to e} \dfrac{-1+\ln x}{\sin(x-e)}$

   (b) $\lim\limits_{x\to(\pi/4)-} (\tan x)^{(1/(-x+\pi/4))}$

5. (12 points)

   a) State the Extreme Value Theorem.

   b) Find the absolute maximum and absolute minimum values of the function $g(x) = x - \sqrt{x}$ on $[0, 4]$, and indicate where it is achieved.

6. (10 points) A particle descends along the linear path $y = 4-2x$ from $(0, 4)$ to $(2, 0)$, where the coordinates x and y are measured in yards. At any time t, let θ denote the angle in radians between the x-axis and the straight line joining the particle to the origin $(0, 0)$. If the particle descends in such a manner that θ decreases at a constant rate of $1/5$ radians per minute, what is the rate of change of the x-coordinate of the particle when $x = 4/3$ yards? Begin by drawing and labeling a picture.
   (Hint: It is helpful to consider $\tan\theta$.)

7. (16 points)

   (a) (5 points) Evaluate $\displaystyle\int \left(\sin x + 3e^x + \frac{1}{x}\right) dx$.

   (b) (5 points) Find the function $h(x)$ for which $h'(x) = -3\sqrt{x} + 6\sec x \tan x$ and $h(0) = 5$.

   (c) (6 points) Let $f(x) = \begin{cases} \sqrt{4-x^2} & \text{if } -2 \le x \le 0 \\ 2-4x & \text{if } 0 < x \le 1 \end{cases}$.

   i) Use geometry to evaluate the integral: $\displaystyle\int_{-2}^{1} f(x)\, dx$.
   First graph $f(x)$ and shade the region between $f(x)$ and the x-axis.

   ii) Does the integral in part i) represent area or only signed area?

END OF EXAM
Please be sure to sign your exam.