Tufts University Department of Mathematics Exam 1

February 24, 2014

You may not use cell phones, books, notes, **or calculators** during the exam. Solutions must be written in your exam book; cross out any work you do not want graded. You are required to **sign your exam book**; with your signature, you pledge that you have neither given nor received assistance on this exam.

Since we have not yet covered it in the lectures, **do not** use l'Hospital's rule to compute limits on this exam.

1. (15 points) Evaluate the following expressions:

(a)
$$\frac{2\ln(6) - \ln(36)}{\ln(7)}$$
 (b) $\sec\left(\frac{11\pi}{6}\right)$

Solve the following equation for x: (c) e^{3x}

(c) $e^{3x} = 3e^{4x}$

Indicate the *domain* of each of the following functions:

(d)
$$f(x) = \ln(1-x)$$
 (e) $g(s) = \frac{1}{\sqrt{s^2-1}}$

2. (8 points) In your blue book, sketch the graphs for the following functions on the indicated domain. Clearly label all points at which the nature of the graph changes as well as all *x*-intercepts and all vertical asymptotes.

- (a) $y = \sin(2x) + 1/2$ on the interval $0 \le x \le \pi$.
- (b) $y = \tan(x/2)$ on the interval $0 \le x \le \pi$.

3. (10 points) Consider the function
$$h(x) = \begin{cases} \frac{4x-7}{4} & \text{for } x \leq 2\\ \frac{1}{x+2} & \text{for } x > 2 \end{cases}$$

- (a) Graph h(x).
- (b) Compute $\lim_{x\to 2^+} h(x)$ and $\lim_{x\to 2^-} h(x)$. Is h(x) a continuous function on the interval $(-\infty,\infty)$? Explain.
- (c) Is h(x) differentiable at x = 2? Explain.
- 4. (10 points) Evaluate the following limits:

(a)
$$\lim_{t \to 4} \frac{2t - 8}{2 - \sqrt{t}}$$

(b)
$$\lim_{z \to \infty} \frac{2 - \cos(2z)}{3z}$$
 (Hint: use the *squeeze theorem*.)

5. (10 points) Consider the function

$$f(x) = \frac{1 + \sqrt{3x^6 + 1}}{x^3 - 1}$$

- (a) Compute $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$. What are the horizontal asymptotes for the graph of f(x)?
- (b) Find all vertical asymptotes of the graph of f(x).

- **6**. (8 points)
- (a) Carefully state the LIMIT DEFINITION for the derivative f'(x) of a function f(x).
- (b) Use the limit definition to find f'(x) when $f(x) = \frac{1}{x+8}$.
- (6 points) Suppose that f(x) is a differentiable function with the properties that

$$f(2) = 3$$
 and $f'(2) = 1$.

Find the following:

(a)
$$\frac{d}{dx}f(2x)\Big|_{x=1}$$
 (b) $g'(2)$ if $g(x) = e^{f(x)}$

8. (18 points) Compute the following: DON'T SIMPLIFY YOUR ANSWERS.

(a)
$$\frac{d}{dx} \left[\frac{x \sin(x)}{x+1} \right]$$

(a)
$$\frac{d}{dx} \left[\frac{x \sin(x)}{x+1} \right]$$
 (b)
$$\frac{d}{ds} \left[(s^{-2} + s^2)^{30} \right]$$

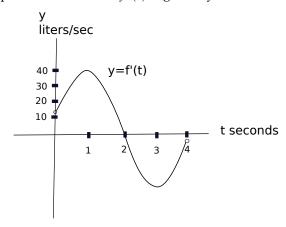
(c) Find
$$\frac{dy}{dx}$$
 at (1,0) if $x^2 + e^{-3y} = 4y + 1 + x$.

9. (7 points) Find the equation of the tangent line to the graph of the function

$$g(x) = 3\tan(x-1) + 2\cos(x-1)$$

at the point (1, g(1)).

10. (8 points) The quantity in liters of water in a reservoir *t* seconds after a certain valve is opened is given by the function f(t). The graph of the derivative f'(t) is given by the following:



Indicate whether the following statements are True or False. No partial credit. No justification required.

- (a) There are 40 liters of water in the reservoir 1 second after the valve is opened.
- (b) There is *more* water in the reservoir at time t = 3 seconds than at time t = 4 seconds.
- (c) The tangent line to the graph of the function f(t) at t = 2 is horizontal.
- (d) The secant line slope $\frac{f(3.1) f(3)}{0.1}$ is positive.

END OF EXAM.