Tufts University

Math 32 | Department of Mathematics | Spring 2014

Exam 2 Review

1. Find $\frac{dx}{dy}$. Do not simplify. Note that some problems will require logarithmic differentiation.

   (a) $y = \cos^{-1}(\sqrt{1+x^2} - x)$

   (b) $y = \ln x + 2^e + 5^x + x^7 + \ln x^{1/2} + \ln 8 + e^{4x}$

   (c) $y = \tan^{-1}\left(\sin(x^2) - \cos(x^2)\right)$

   (d) $y = (5x+2)^{\cos x}$

   (e) $y = \dfrac{(7x-2)^4(x^2+5)^{17}}{(4x^3+9x)^6}$

2. Find the domain of each function.

   (a) $g(x) = \dfrac{\sqrt{e^x+1}}{\ln(x+1)-1}$

   (b) $h(x) = \ln(x+3)(x+1)(2-\sqrt{x})$

3. Find all critical points of each function.

   (a) $f(x) = \sqrt[3]{x^2 - x}$

   (b) $f(x) = (\sin^2 x) - x$

   (c) $f(x) = |2x - 1|$

4. i) State the Extreme Value Theorem.
   ii) What are the possible candidates for the absolute maximum and absolute minimum of a continuous function on a closed interval?
   iii) Find the absolute maximum value of each function on the given interval, and indicate where it is achieved.
   (a) $f(x) = x^3 - 6x^2 + 9x + 1$ on $[2, 4]$
   (b) $g(x) = \dfrac{x^2-9}{x^2+9}$ on $[1, 4]$
   (c) $h(x) = \sqrt[3]{x}(8-x)$ on $[0, 8]$
   (d) $f(x) = e^{-x} - e^{-2x}$ on $[0, \ln 9]$
   (e) $g(x) = \dfrac{\ln x}{x^2}$ on $[1, e]$

5. Find all the x-coordinates of all local minima and local maxima of each function below, indicating whether it is a local maximum or local minimum.

   (a) $f(x) = 3x^3 - 18x^2 + 5$

   (b) $f(x) = \dfrac{x}{x^2-9}$

   (c) $f(x) = \dfrac{x^2}{x^2+9}$

   (d) $f(x) = \sin x - \cos x$ on $[0, 2\pi]$

6. Two cars start from the same point: the first car travels east at 25 mph, while the second car travels north at 35 mph. How fast is the distance between the two cars changing 4 hours later?

7. When a cylinder has radius 1 inch and height 5 inches, the volume of the cylinder is increasing at 10π in^3/second and its radius is increasing at 3 in/second. How fast is the cylinder's height changing?

8. A paper cup has the shape of a cone with height of 10 cm, and a radius at the top of 3 cm. If water is poured into the cup at a rate of 2 cm ^{3}/second, how fast is the water level rising when the water is 5 cm deep?

9. A particle descends along a circular path of radius 12 from $(0, 12)$ to $(12, 0)$, where the coordinates x and y are measured in feet. At any time t, let θ denote the angle in radians between the x-axis and the straight line joining the particle to the origin $(0, 0)$. If the particle descends in such a manner that y decreases at a constant rate of 1 foot per minute, what is the rate of change of θ when y is 6 feet? Begin by drawing and labeling a picture. (Hint: Express y in terms of θ.)

10. Find each limit. If you use L'Hopital's rule, indicate which kind of indeterminate form is involved.

    (a) $\lim_{x\to\infty} x^{-3}e^x$

    (b) $\lim_{x\to\infty} \frac{\ln\ln x}{x}$

    (c) $\lim_{x\to 1} \frac{\ln x}{\sin \pi x}$

    (d) $\lim_{x\to 0} \frac{1-e^x}{\cos(x)}$

    (e) $\lim_{x\to 0^+} (\tan x)^x$

    (f) $\lim_{x\to\infty} \left(1+\frac{4}{x}\right)^x$

    (g) $\lim_{x\to 0^+} \left(\frac{1}{x}+\ln x\right)$

11. Given

$$f(x) = \frac{18(x+1)}{(x+3)^2} \qquad f'(x) = \frac{18(1-x)}{(x+3)^3} \qquad f'' = \frac{36(x-3)}{(x+3)^4},$$

    answer the following questions about the graph $y = f(x)$.
    (a) Find the $x-$ and $y-$intercepts.
    (b) Find any horizontal and/or vertical asymptotes. Compute the appropriate limits to justify your answers.
    (c) Find the intervals where f is increasing and where f is decreasing.
    (d) Find the $x-$ and $y-$coordinates of any local minimums or local maximums.
    (e) Find the intervals where f is concave up and where f is concave down.
    (f) Find the $x-$ and $y-$coordinates of any inflection points.
    (g) Sketch the graph of $y = f(x)$, indicating the features you have found above (ie, label the $x-$ and $y-$intercepts, any asymptotes, local min/max points, and inflection points.

12. Integration related problems from the book to work which have answers in the back of the book.

    a) Indefinite integrals: p. 304 51, 53, 55, 57, 59, 61
    b) Initial value problems: p. 301 43, 47
    c) Definite integrals to evaluate geometrically: p. 332 25, 27, 29;
    p. 333 67 (find area and net area), 71;
    p. 367 3, 5 (Does each definite integral represent area or only net area?)
    d) Definite integrals to evaluate using the Fundamental Theorem of Calculus: p. 348 77, 79, 81;
    75 (Does this definite integral represent area or only net area?)