Math 226 | Numerical Analysis | Fall 2013

Course Information

BLOCK: J+: 3:00p-4:15p, Tuesday and Thursday
INSTRUCTOR: Kye Taylor
EMAIL: `kye.taylor@tufts.edu`
OFFICE: Bromfield-Pearson 217
OFFICE HOURS: Tuesday and Thursday 10a-12p, Friday 10a-11a, and by appt.
COURSE PAGE: http://courses.math.tufts.edu/math226

PREREQUISITES:

- Experience in a scientific programming language (e.g. Fortran, C, C++, MATLAB, Python, etc.), though the expectation will be to program in MATLAB, C++, or Python. Training sessions will be held if necessary.
- Familiarity with linear algebra, differential equations, and real analysis, all at the undergraduate level.

RECOMMENDED TEXT:

- "An Introduction to Numerical Analysis," by Atkinson, 2nd edition, Wiley.
  DO NOT confuse this with a textbook by Atkinson and Han.

BRIEF DESCRIPTION: "Numerical analysis is the study of algorithms for the problems of continuous mathematics." (L. N. Trefethen, "The definition of numerical analysis", SIAM News, November 1992.) To develop the algorithms and study their behavior, we will need a number of tools from calculus/analysis/linear algebra, as well as an understanding of floating point arithmetic on a computer and the potential sources of error that may be propogated during program execution.

This course is intended as a rigorous first graduate-level course in Numerical Analysis. We will cover all of the Numerical Analysis qualifying exam core topics, which include numerical integration and orthogonal polynomials, interpolation, numerical solution of differential equations, unconstrained optimization, and solution of nonlinear equations.

LEARNING OBJECTIVES: Learning objectives relevant to a Master's Degree or a Ph.D. in mathematics are available at

http://ase.tufts.edu/faculty/committees/objectives/math.htm

Learning objectives relevant to the qualifying exam on Numerical Analysis are available at

http://math.tufts.edu/graduate/qualifyingExams.htm

HOMEWORKS: Typical homework assignments will include problems that can be worked through by hand, as well as programming tasks that must be coded on a computer using MATLAB, C++, or Python. If you prefer to program in another language, please speak with me as soon as possible.

For each homework assignment, please provide a write-up that details the mathematical manipulations required for those problems completed by hand. If applicable, the write-up should also address any homework problems that are related to the execution or design of relevant code.

Homework problems that can be worked through by hand will be collected in class. Please mark your homework with the course and section numbers as well as an identifier to help you know that it is yours – something that is likely unique to your section and something that is pronounceable in case the homework is returned by calling out the identifiers. Please write it as clearly as possible and make sure to tell your instructor well before the end of the semester what your identifier is so credit associated with it can be counted towards your course grade.

Feel free to use your name as your identifier, but expect that unless you are told otherwise, the homework will be handed off between instructor and grader in a way that does not ensure their confidentiality (usually by way of drawers in the lobby of the Bromfield-Pearson building). Your educational record is privileged information under the federal Family Educational Rights and Privacy Act (FERPA), and using your name as identifier means that you opt out of being guaranteed the confidentiality of the information on and in your homework.

Any programs that you are asked to code will also be collected via email. Specific instructions for email-submissions of homework will be given in the homework assignment itself.

DISABILITY SERVICES: If you are requesting an accommodation due to a documented disability, you must register with the Disability Services Office at the beginning of the semester. To do so, call the Student Services Desk at 617-627-2000 to arrange an appointment with Linda Sullivan, Program Director of Disability Services.

GRADES:

2 Quizzes	=	15%
1 Takehome Midterm	=	25%
8 Homeworks	=	25%
1 Takehome Final	=	35%

CALENDAR:

What	Assigned	Due
HW 1	Tues. Sept. 3	Tues. Sept. 10
HW 2	Tues. Sept. 10	Tues. Sept. 17
HW 3	Tues. Sept. 17	Tues. Sept. 24
In-class Quiz	Tues. Oct. 1	
HW 4	Tues. Oct. 1	Tues. Oct. 8
HW 5	Tues. Oct. 8	Fri. Oct. 18
Midterm	Fri. Oct. 17	Fri. Oct. 25
HW 6	Fri. Oct. 25	Fri. Nov. 1
HW 7	Fri. Nov. 1	Fri. Nov. 8
In-class Quiz	Thur. Nov. 14	
HW 8	Fri. Nov. 15	Tues. Nov. 26
Final*	Tues. Dec. 3	Fri. Dec. 13

SEMESTER OUTLINE:

- Preliminaries
  - Relevant theorems from Calculus
  - Relevant norms
  - Floating point numbers
  - Computer arithmetic
  - Definitions and sources of error
  - Definitions of stability and well-posedness
- Linear Systems of Equations (Parts of chapter 7 and 8)
  - Gaussian Elimination with partial pivoting (equivalence to $AP = LU$)
  - Operation counts
  - LDL^T and Cholesky for symmetric matrices
  - SVD, condition numbers, and matrix norms
- Nonlinear Systems of Equations
  - 2.1 Bisection (1D)
  - 2.2 Newton's method
  - 2.3 Secant method
  - 2.5 Fixed point iteration
  - 2.10 Systems of nonlinear equations
  - 2.11 Newton's method for systems, and alternatives
- Optimization
  - 2.12 Unconstrained optimization

    Quasi-Newton
    Nonlinear least-squares

- Numerical Interpolation
    - 3.1 Polynomial interpolation theory
    - 3.6 Hermite interpolation
    - 3.7 Piecewise polynomials (cubic splines)
    - B-splines
    - 4.1 Weierstrass approximation theorem
    - 4.4 Orthogonal polynomials
    - 4.5 Least-squares approximation via orthogonal polynomials
- Numerical Integration
    - 5.1 Trapezoidal and Simpson's rules
    - 5.2 Newton-Cotes Integration
    - 5.3 Gaussian quadrature
    - 5.4 Euler-Maclaurin summation theorem, Richardson extrapolation
    - Adaptive quadrature
    - 5.7 Numerical differentiation, Undetermined coefficients
    - Taylor series in numerical differentiation
- Numerical Differential Equations
    - 6.1 - 6.5 Euler's, implicit, explicit, trapezoidal, and midpoint methods
    - 6.9 Stiff differential equations and the method of lines