Math 21-01	Introductory Statistics Midterm 2	Fall 2015 November 13th 2015

NAME: Midterm 2 Solutions

HONESTY STATEMENT: My signature below indicates that I neither gave nor received assistance from any person or source, other than those specified in the instructions given below. All violations will result in an F in the course and you will be reported to the appropriate Dean.

Signature: xxxxxxxxxxxxxxxxxxxxxxx

INSTRUCTIONS: Please mark your answers on the exam in the space provided. You are not allowed to use any calculators, books, or notes. A list of formulas and tables are included in the last few pages of the exam. Answers given on the exam can be left as unsimplified formulas when appropriate. If your answer space is messy/difficult to read, please circle your final answer.

SCORE:

Problem 1:	10	(out of 10)
Problem 2:	15	(out of 15)
Problem 3:	10	(out of 10)
Problem 4:	15	(out of 15)
Bonus:	5	(Maximum 5 points)
Total:	55	(out of 50)

PROBLEM 1:

For the following situations, please write the critical value(s) to test the given hypothesis, and from what distribution they are taken from.

If you have $n = 15$ and want to test the hypothesis $H_1 : \sigma \neq 3$ with $\alpha = 0.05$.
5.629 and 26.119 from Chi-Squared

If you have $n = 20$ and want to test the hypothesis $H_1 : \mu \neq 55$ with $\alpha = 0.01$ and $\sigma = 10$.
-2.575 and 2.575 from Normal

If you have $n = 35$ and want to test the hypothesis $H_1 : p < 1/2$ with $\alpha = 0.01$.
-2.235 from Normal

If you have $n = 15$ and want to test the hypothesis $H_1 : \mu \neq 14.5$ with $\alpha = 0.05$.
-2.145 and 2.145 from Student t

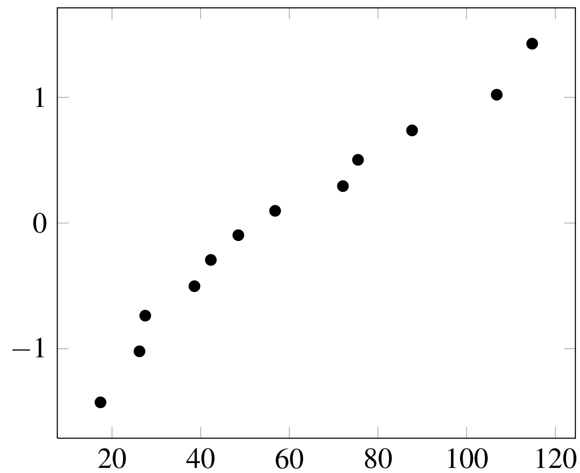
If you have $n = 20$ and want to test the hypothesis $H_1 : \sigma^2 > 2.5$ with $\alpha = 0.05$.
30.144 from Chi-Squared

PROBLEM 2:

For this problem, consider the following sample of domestic box office gross (in millions, adjusted for inflation) for a simple random sample of 12 horror movies:

114.8 56.8 87.7 75.5 48.5 42.3 38.6 27.5 26.2 17.4 106.8 72.1

The sample mean is $\overline{x} = 59.5$ and the sample standard deviation $s = 32.0$.
The Normal Quantile Plot for our sample is constructed below.



Does the data satisfy our requirements for estimating μ? Why or why not?
Yes. Even though we do not have $n \geq 30$, our NQP is reasonably close to a straight line without an obvious pattern, which means we can reasonably assume our data is from a normally distributed population.

Some people felt that the NQP systematically deviated from a straight line and thus our data is from a uniformly distributed population, and thus our requirements for estimating μ were not satisfied. If you jutified it, I accepted this answer.

Which distribution would we use to estimate the population mean in this case?
Since we do not know σ, we must use the Student t distribution.

Write an expression for a 95 % confidence interval to estimate the population mean.
$\overline{x} - t_{\alpha/2}\frac{s}{n} < \mu < \overline{x} + t_{\alpha/2}\frac{s}{n}$
$59.5 - 2.201\frac{32.0}{\sqrt{12}} < \mu < 59.5 + 2.201\frac{32.0}{\sqrt{12}}$

Using a calculator, I find that our 95 % confidence interval is $39.17 < \mu < 79.83$. If a studio executive claims that horror movies have a mean domestic box office of $80 million. Can we use the confidence interval to test this hypothesis? If so, what is our setup and what would we conclude?
We can, because for means, 95% CI is equivalent to a two-tailed hypothesis test with $\alpha = 0.05$. Our setup is $H_0 : \mu = 80$, $H_1 : \mu \neq 80$. Since 80 is not in our confidence interval, we reject the null hypothesis.

If I tell you that the sample is actually made up of the twelve movies in the "Friday the 13th" movie franchise, how does this change things?
The sample is not a simple random sample, and so our methods can not apply!

PROBLEM 3:

A small boat can safely carry a weight of $1,800$ lbs, and is being loaded with passengers from a population with normally distributed weights with a mean of 170 lbs and standard deviation 30 lbs. If the boat is loaded with nine passengers, what is the maximum safe mean weight of passengers?

$1800/n = 1800/9 = 200$

What is the probability of a single passenger exceeding the maximum safe mean weight?

First find z-score: $Z = \frac{200-170}{30} = 1$. Then look on the table for the z-score of 1, and take its complement. $P(x > 200) = P(z > 1) = 1 - P(z \leq 1) = 1 - .8413 = .1587$.

What is the probability that the total load of nine passengers exceeds the safe limit of $1,800$ lbs?

First find z-score: $Z = \frac{200-170}{(30/\sqrt{9})} = \frac{200-170}{10} = 3$. Then look on the table for the z-score of 3, and take its complement. $P(x > 200) = P(z > 3) = 1 - P(z \leq 3) = 1 - .9987 = .0013$.

Let n be the number of passengers we are loading into the boat. We want to find the maximum number of passengers to allow while having the probability of overloading at most 0.01. Write the expression that we would need to solve to find this value.

Judging by our table, the Z-score corresponding to 0.99 area to the left is 2.325. Thus, we need to solve

$$2.325 \leq \frac{\frac{1800}{n} - 170}{\frac{30}{\sqrt{n}}}.$$

We would use a computer to find when the they are equal, and and round down to the nearest whole number.

PROBLEM 4:

I claim that $1/5$th of the population is left handed. However, you think that the true value is much lower than that. To test this, you construct a simple random sample of 30 adults, and observe that only two of them are left handed. We seek to do a hypothesis test with $\alpha = 0.05$. Under this setup, what are the following:

$H_0 : p = 1/5$

$H_1 : p < 1/5$

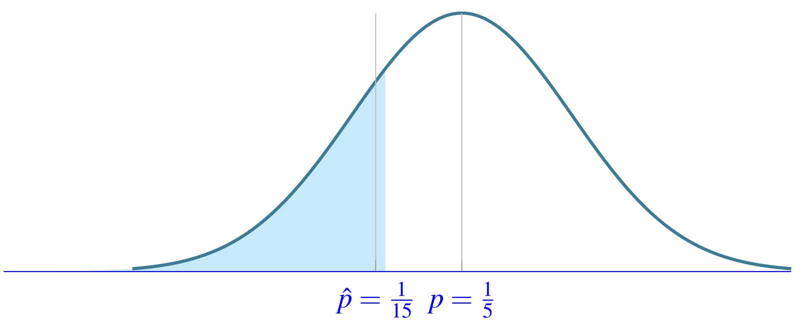
Critical Value(s) = -1.645

$\hat{p} = 1/15$

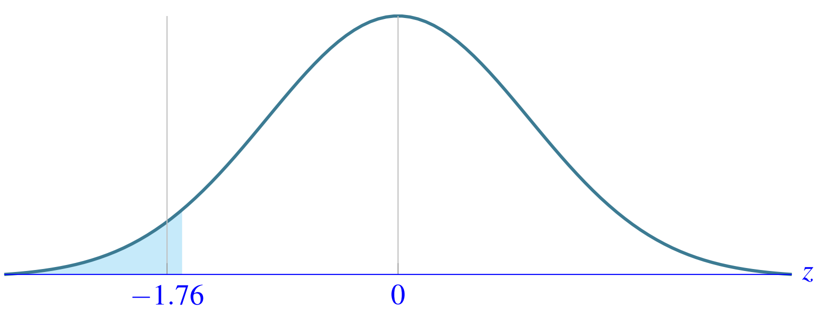
Are the requirements for hypothesis testing satisfied?
Yes, We have a simple random sample, and $np = 6 \geq 5$ and $nq = 24 \geq 5$.

After some careful rounding, we compute the test statistic to be -1.76.
Draw the sampling distribution, the critical region, and our test statistic.

Sampling Distribution:



Normalized:



There was an ambiguity as to which graph I was asking for, so I accepted either.
In the first, our point estimate $1/15$ sits inside of our critical region, which is the left tail.
In the second, our test statistic -1.76 sits inside of the critical region, which is again the left tail.

If possible, calculate the P-value. If not, find a range for the P-value.

Since this is one-tailed, our P-value is .0329

Explain in terms of the critical value method *and* P-value method the conclusion of our hypothesis test.

Critical values: since $-1.76 < -1.645$, we reject the null hypothesis.

P-values: since $.0329 < 0.05$, we reject the null hypothesis.

If instead of $\alpha = 0.05$, we had chosen $\alpha = 0.01$, would our conclusion have been the same?

No. $.0329 > 0.01$, so we would fail to reject the null hypothesis.

BONUS:

The P-value in problem 4 can be computed using exact methods. What distribution would we use to do so? Using this distribution, write (but do not calculate) the exact expression for P.

We could do so using the Binomial distribution. If X is the discrete random variable counting successes with $p = 1/5$, then our P-value is exactly

$$P(X \leq 2) = \sum_{i=0}^{2} {}_{30}C_i \left(\frac{1}{5}\right)^i \left(\frac{4}{5}\right)^{30-i}$$

$$= \left(\frac{4}{5}\right)^{30} + 30\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^{29} + \frac{(30)(29)}{2}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^{28}$$