Chapter 22 homework solutions

1) Parents: Custody, home, vacation house, cash

<table>
<thead>
<tr>
<th></th>
<th>Custody</th>
<th>Home</th>
<th>Vacation House</th>
<th>Cash</th>
</tr>
</thead>
<tbody>
<tr>
<td>John</td>
<td>0.50</td>
<td>0.15</td>
<td>0.15</td>
<td>0.20</td>
</tr>
<tr>
<td>Jane</td>
<td>0.15</td>
<td>0.45</td>
<td>0.10</td>
<td>0.30</td>
</tr>
</tbody>
</table>

ratio: 10/3 1/3 3/4 2/3

If r = 1, John gets 0.5 + 0.15 = 0.65
Jane gets 0.45 + 0.3 = 0.75

So move r to the right, so r = 2/3, and they will split the cash.

Let \( p = \) fraction of cash John gets. \( 1 - p = \) fraction of cash Jane gets.

\[
\begin{align*}
0.5 + 0.15 + p(\frac{1}{3}) &= 0.45 + (1-p)(\frac{1}{3}) \\
0.65 + 0.2p &= 0.45 + 0.3 - 3p \\
0.5p &= 0.1 \\
p &= \frac{1}{5}
\end{align*}
\]

John gets custody, vacation house and 1/5 of the cash.
Jane gets 4/5 of the cash and the home.

Check: John gets \( 0.5 + 0.15 + \frac{1}{5}(\frac{1}{3}) \) = 0.65 + 0.04 = 0.69
Jane gets \( 0.45 + \frac{4}{5}(\frac{1}{3}) \) = 0.45 + 0.24 = 0.69

2 a) Car Bank Total

<table>
<thead>
<tr>
<th></th>
<th>Car</th>
<th>Bank</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>John</td>
<td>6000</td>
<td>20000</td>
<td>26000</td>
</tr>
<tr>
<td>Jane</td>
<td>4000</td>
<td>20000</td>
<td>24000</td>
</tr>
</tbody>
</table>

John 3/13, 10/13
Jane 1/6, 5/6
ratio 17/13, 12/13

\[
\frac{3}{13} + p\left(\frac{10}{13}\right) = \left(1-p\right)\left(\frac{5}{6}\right)
\]

Multiply both sides by 613

\[
\begin{align*}
18 + 60p &= 5(13)(1-p) \\
18 + 60p &= 65 - 65p \\
135p &= 47 \\
p &= \frac{47}{135}
\end{align*}
\]

John gets car and \( \frac{47}{135} \) (bank) = car and \( \frac{47}{125} \) (20000) = 57520, and car for John
Jane gets \( \frac{78}{125} \) (bank) = \( \frac{78}{125} \) (20000) = 5124.80, for Jane
1a) continued

Check that it is equitable:

John gets \[ \frac{\$4000 + \$7520}{\$26000} = 52\% \]
Jane gets \[ \frac{\$12480}{\$24000} = 52\% \]

\[ \begin{align*}
\text{John gets} & \quad \frac{3}{13} + \frac{47}{125} \cdot \frac{10}{13} = \frac{3}{13} + \frac{47}{25} \cdot \frac{2}{13} \\
& = \frac{75 + 94}{25 \cdot 13} = \frac{169}{25 \cdot 13} = \frac{13}{25} \\
\text{Jane gets} & \quad \frac{5}{6} \cdot \frac{78}{125} = \frac{1}{6} \cdot \frac{78}{25} = \frac{13}{25}
\end{align*} \]

2b) If each put \$10,000 into the bank account, then there is \$20,000 in the bank, and the adjusted winner method proceeds as in part a above. If we subtract out the \$10,000 each put in, Jane gets \(12480 - 10000 = 2480\), John "gets" \(7520 - 1000 = 2480\). The net result is that John gets the car, and pays Jane \$2480.