Incorrect Proofs that Pairwise Comparison is Unanimity Fair

Find the errors in these proofs from former students

Proof 1:

- Assume in a preference schedule, everyone prefers X over Y so X wins in pairwise points.

  X : Y = N : 0

- If a third candidate, Z is introduced, and Y beats Z, that means X will also beat Z.

  Say Y gets j pairwise points so X would get at least j + 1 pairwise.

- Y wouldn't have enough pairwise points to beat X so X wins.

Proof 2:

If all voters chose X over Y then X beats Y head to head. If Y beats Z , then X will either beat or tie Z . Therefore no matter how many pairwise points awarded to Y = j X will always have at least j + 1 points.

Proof 4:

- If X > Y, then X : Y is N : 0
- This means that all voters prefer X : Y on all ballots

Assume a new candidate Z joins

- If Y beats Z in 1 - 1, then X also beats Z in 1 - 1
- If Y gets j pairwise points from Z , then X must get at least j + 1 pairwise points

  j + 1 > j , so Y $\notin$ W

Proof 8:

Pairwise Comparison is unanimity fair because if A is ranked above B in every ballot it will win the one to one comparison. If B is above C ~~then~~ on every ballot, then A will be above C on every ballot. A will thus get at least the same number of Pairwise points as C plus one, ensuring A beats C.

Proof 18:

If A : B is the 1 : 1 comparison, and A : B is N : 0, meaning that A is always above B. Because pairwise assigns points based on 1 : 1 comparisons, A's points, call them j, will always be greater than B's, k. Since j > k, A will be a member of the winning set.

Proof 11:

- X beats Y in all head-to-head competitions, and Y beats Z in all head-to-head competitions.
- This means that X also beats Z in all head-to-head competitions.
- This means that for every point or half-point that Y receives, X also receives the same.
- Additionally, X receives the point from beating Y, meaning that X has at least one point more than Y .
- This ensures that Y is not among the set of winners, thus upholding unanimity criterion.

Proof 13:

If all voters prefer X over Y , then X : Y is N : 0, and
X is above Y in all ballots.
Now consider candidate Z.
If Y beats Z in a (1-1) comparison, then X also beats
Z with the same or greater margins.
So if Y gains j points, then X also gains j points
but X also beats Y in (1-1) comparison so he
gains at least j + 1 points. Since j + 1 > j, $Y \notin W$ and $X \in W$
Therefore, pairwise is unanimity fair.

Proof 14:

- Suppose the 1 - 1 comparison of [X : Y] = [N : 0]
- Now suppose that Y beats candidate Z in 1-1 comparison.
- Y gets at least 1 pairwise point for beating Z.
- Because X beats Y , he also beats Z.
- Therefore X must earn at least 2 pairwise points, one for beating Z, and one for beating Y.
- Therefore $Y \notin \{W\}$

Proof 15:

- If candidate X is preferred over candidate Y in the preference schedule
- If candidate Y is preferred over candidate Z, then X is also preferred over Z
- If Y earns j pairwise points, X will also earn j pairwise points
- Since X is preferred over Y, X will earn at least j + 1 pairwise points
- Since J + 1 > j , Y can't be included in the set of winners.