

Non-comprehensive Review Problems for Exam 1

- 1) Suppose 3 people are dividing a plot of land using the Steinhaus method, and the values A and C gives to each piece is summarized in the following table:

	S_1	S_2	S_3
A	$\frac{1}{2}$	0	$\frac{1}{2}$
B			
C	$\frac{1}{3}$	0	$\frac{2}{3}$

- a) Fill in the row of values for B.
 b) What are the bid lists for each player?
 c) Find a fair and envy-free division.

d) Is the division you found in part c equitable? Why?

e) Find a fair division in which there is envy, and specify the envy.

- 2) Suppose 4 people are dividing a plot of land, and the values each person gives to each piece is summarized by the following table:

	S_1	S_2	S_3	S_4
A	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
B	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$
C	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$
D	0	0	$\frac{5}{6}$	$\frac{1}{6}$

- a) What are the bid lists for each player?
 b) Find a fair division with A getting S_1 .

c) Is the division you found in part b equitable?

d) What is the least B can get in the division you found in part b?

e) What is the least D can get in the division you found in part b?

f) Is the division you found in part b envy-free?

- 3) Three people are using Steinhaus to divide a berry tart with the configuration below. Suppose A likes all types equally, B likes L and M equally, but likes H twice as much as each of those, and C likes H and L equally, but hates M.

L	L	L
H	L	L
H	M	L
H	M	M

- (a) What is the total value of the tart for each player? (this can just be an integer)

- (b) What is a fair share for each player?

Suppose A divides the tart along the columns, and we call the three pieces *left*, *middle*, and *right*.

- (c) What are the bid lists for each player?

- (d) Find the fair division that results.

- (e) Fill out the following grid to describe what each player thinks everyone got.

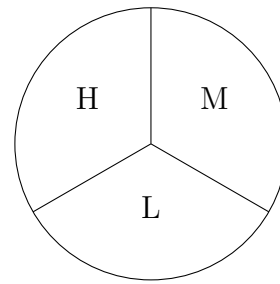
	A received	B received	C received
A thinks			
B thinks			
C thinks			

- (f) List any instances of envy in the division.

- (g) Is this division equitable?

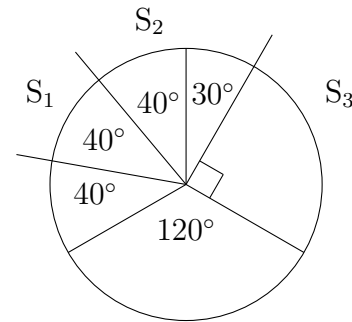
4) Suppose 3 people (A, B and C) are using Selfridge/Conway to divide a pie that has equal parts Huckleberry, Mulberry and Loganberry, and their preferences are summarized in this table:

	H	M	L
A	1	0	0
B	x	x	x
C	x	0	x



round 1:

Suppose A divides pie into the three pieces pictured.
Find the value of each piece to B.



$S_1 =$

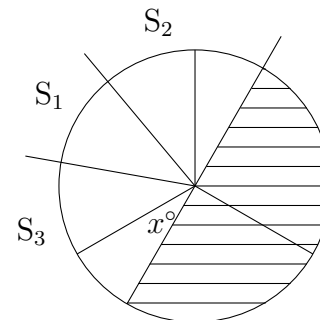
$S_2 =$

$S_3 =$

Suppose B makes the trimmings as shaded,
So the trimmed S_3 has 40° of H and x° of L.

What should be the value of angle x ?

Find the value of each piece to C.



$S_1 =$

$S_2 =$

trimmed $S_3 =$

In what order do the players now choose pieces, and which piece does each get?

Complete this table to show what each person gets (as a fraction of the whole) at the end of round 1:

round 1	A gets	B gets	C gets
A thinks			
B thinks			
C thinks			

Who has a fair share so far?

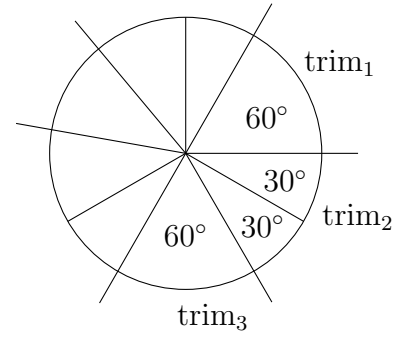
Is there any envy so far?

round 2:

Who divides the trimmings?

Suppose the trimmings are divided as pictured.

Now to complete round 2, in what order do they choose?



Who gets which piece?

Complete this table to show what each person gets (as a fraction of the whole) at the end of round 2:

total of both rounds	A gets	B gets	C gets
A thinks			
B thinks			
C thinks			

Is this fair?

Is this envy-free?

Is this equitable?

Is this Pareto-optimal?

5) In round two of Selfridge/Conway, the trimmings are divided into three pieces by U. Then they players choose their shares of the trimmings in the order T, A, U. This is the only order of dividing and choosing that will guarantee an envy-free outcome. Now consider the following alternate possibility for dividing and choosing:

U divides, A chooses, T chooses, then U gets the last piece.

a) Could U have envy? Why or Why not?

b) Could A have envy? Why or Why not?

c) Could T have envy? Why or Why not?

6) Suppose two people are dividing a pie with four components, and the table below lists the values each player has for each type of pie. Compute the valuation ratios for each type of pie to complete the table:

	G	H	L	M
A	.2	.4	.2	.2
B	.3	.2	.4	.1
$\frac{a_j}{b_j}$				

Set up the dots, let $r=1$, and describe the threshold division that results.

Is this division Pareto-optimal?

Is it equitable? Why or why not?

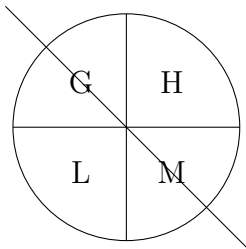
Using the Brams/Taylor Adjusted Winner method, move r so A and B will share a component, let p be the fraction of the shared component that A will get, and find the equitable arrangement.

Find the fraction of the whole pie that each person gets with this division.

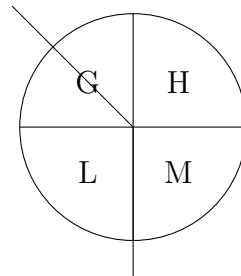
Is this division Pareto-optimal?

Is it equitable? Why or why not?

Continuing with the example above, could the following divisions be Pareto optimal?



yes or no?



yes or no?