Determine whether the following series is absolutely convergent, conditionally convergent, or divergent. Justify your answer. State and check hypotheses of any test, rules or theorems you use.

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{400 + n^2} \]

\[ \sum_{n=1}^{\infty} \frac{n}{400 + n^2} = \sum_{n=1}^{\infty} \frac{\frac{n}{n^2}}{\frac{400}{n^2} + 1} \leq \sum_{n=1}^{\infty} \frac{1}{n^2} \]

Let \( L = \lim_{n \to \infty} \frac{n}{400 + n^2} = \lim_{n \to \infty} \frac{n}{400 + n^2} = \lim_{n \to \infty} \frac{1}{400/n^2 + 1} = 1 \)

Since \( 0 < 1 < n \) and since \( \sum \frac{1}{n} \) is the divergent harmonic series (\( p \)-series, \( p=1 \leq 1 \)) our series is not \( \text{ABC} \).

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{400 + n^2} \]

\[ \text{AST} \]

\[ \lim_{n \to \infty} \frac{n}{400 + n^2} = \lim_{n \to \infty} \frac{n}{400 + n^2} = 0 \]

\[ \frac{n}{400 + n^2} > 0 \text{ for } n > 1 \]

\[ \text{Show } \frac{n}{400 + n^2} \text{ is eventually decreasing;} \]

Let \( f(x) = \frac{x}{400 + x^2} \), \( x > 1 \)

\[ f'(x) = \frac{(400 + x^2) - x(2x)}{(400 + x^2)^2} = \frac{400 - x^2}{(400 + x^2)^2} < 0 \text{ for } x > 20 \]

\[ f(x) \text{ is decreasing for } x > 20 \text{ so } \frac{n}{400 + n^2} \text{ decreases for } n > 20 \]

So our series is conditionally convergent.