Determine whether the following series converges or diverges. Justify your answer. State and check hypotheses of any test, rules or theorem you use.

$$\sum_{n=2}^{\infty} \frac{1}{n \sqrt{\ln n}}$$  

**Integral Test**  

Let $$f(x) = \frac{1}{x \sqrt{\ln x}}, \ x \geq 2$$

Then:

1. $$f(x)$$ is continuous and positive for $$x \geq 2$$.
2. Need to show $$f$$ is eventually decreasing.

$$f(x) = (x \sqrt{\ln x})^{-1}$$

$$f'(x) = -1 \left( x \sqrt{\ln x} \right)^{-2} \left[ \frac{1}{\sqrt{\ln x}} + x \cdot \frac{1}{2} (\ln x)^{-\frac{1}{2}} \cdot \frac{1}{x} \right] \leq 0 \quad \text{for } x \geq 2$$

Since $$f'(x) < 0$$ for $$x \geq 2$$, $$f$$ is decreasing for $$x \geq 2$$.

**Option 2**

Since $$x, \sqrt{\ln x}$$ are both positive and increasing, $$x \sqrt{\ln x}$$ is increasing, so $$\frac{1}{x \sqrt{\ln x}}$$ is decreasing for $$x \geq 2$$.

So we can use the integral test:

$$\lim_{t \to \infty} \int_{2}^{t} \frac{1}{x \sqrt{\ln x}} \, dx = \lim_{t \to \infty} \left[ \frac{2}{\ln x} \right]_{2}^{t} = \lim_{t \to \infty} \frac{2}{\ln t} - \frac{2}{\ln 2}$$

$$= \lim_{t \to \infty} \left( \text{"form" } \int \frac{du}{u^{2}} = \int u^{-2} \, du = \frac{1}{u} + C \right) = 2 \sqrt{\ln t} + C$$

Since the integral diverges, the series diverges.