No books, notes, or calculators. TURN OFF YOUR CELL PHONE. ANYONE CAUGHT WITH THEIR CELL PHONE ON WILL BE GIVEN A 10 POINT DEDUCTION. Cross out what you do not want us to grade. You **must** show work to receive full credit. Please try to write neatly. You need not simplify your answers unless asked to do so. You should evaluate standard trigonometric functions like \( \tan(\pi/3) \). You are not allowed to quote results about growth rates. You are required to **sign** your exam book. With your signature, you pledge that you have neither given nor received assistance on this exam.

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1. (12 points) Integrate.

(a) \[ \int \cos^3(x) \, dx \]

(b) \[ \int \frac{dx}{x^2 - 3x - 4} \]
2. (8 points) Find the limits of each of the following sequences or state that they do not exist.

(a) \( a_n = \frac{5\sin n}{n^3} \)

(b) \( a_n = n \ln \left(1 + \frac{4}{n}\right) \)
3. (12 points) Determine whether each of the following series converges or diverges. Justify your answer. State and check hypotheses of any test, rules or theorems you use. You may not simply quote a theorem.

(a) \[ \sum_{k=1}^{\infty} \frac{24k^2 + 30k}{k^3 + 1} \]

(b) \[ \sum_{k=1}^{\infty} \frac{(-1)^k k}{10e^k} \]
4. (7 points) Find the radius of convergence and interval of convergence for the following power series:

\[
\sum_{k=1}^{\infty} \frac{(-1)^{k+1}(x - 6)^k}{4^k \cdot k}
\]
5. (8 points) Compute the Taylor series for
\[ f(x) = \frac{1}{(x + 3)^2} \]
centered at \( a = -2 \) using the definition of the Taylor series. Write the series using summation notation. (You do not need to find the radius of convergence or the interval of convergence.) Simplify your answer.
6. (8 points) Consider the following parametric equations:

\[ x = e^t \] and \[ y = 4 - e^{2t} ; \quad -\infty < t \leq \ln 2 \]

(a) Eliminate the parameter \( t \) to obtain an equation in \( x \) and \( y \).

(b) Sketch the curve and indicate the positive orientation.
7. (6 points) Sketch each of the following sets of points in the polar plane. (Use separate sketches for each set.)

(a) \{ (r, \theta) : |r| \geq 2 \text{ and } \frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4} \} \\

(b) \{ (r, \theta) : 0 < r \leq 3 \text{ and } \frac{2\pi}{3} \leq \theta \leq \frac{5\pi}{3} \}
8. (8 points) Polar Curves

(a) A Cartesian and a polar graph are given. Identify the points on the polar graph that correspond to the points shown on the Cartesian graph.

i. Point A on the Cartesian graph corresponds to point ___ on the polar graph.

ii. Point C on the Cartesian graph corresponds to point ___ on the polar graph.

iii. Point B on the Cartesian graph corresponds to point ___ on the polar graph.

(b) The following is the graph of a polar curve drawn in the Cartesian plane. Draw the curve in the polar plane next to it.
9. (12 points) Complex numbers

(a)  
   i. Write \((-\sqrt{3} + i)\) in polar form.

   ii. Find \((-\sqrt{3} + i)^6\) and write your answer in rectangular form.

(b)  
   i. Find the cube roots of \(z = -8\).

   ii. Plot the roots in the complex plane.
You may use any of the following for the rest of the exam.

\[
\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^k + \cdots = \sum_{k=0}^{\infty} x^k, \quad \text{for } |x| < 1
\]

\[
\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + \frac{x^k}{k} + \cdots = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}x^k}{k}, \quad \text{for } -1 < x \leq 1
\]

\[
e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^k}{k!} + \cdots = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \quad \text{for } |x| < \infty
\]

\[
sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{(-1)^{k+1}x^{2k+1}}{(2k+1)!} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, \quad \text{for } |x| < \infty
\]

\[
\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{(-1)^k x^{2k}}{(2k)!} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}, \quad \text{for } |x| < \infty
\]

10. (5 points) Evaluate the following limit using series:

\[
\lim_{x \to 0} \frac{2 \cos 2x - 2 + 4x^2}{2x^4}
\]

11. (6 points) Identify each of the functions represented by the following power series.

(a) \[
\sum_{k=0}^{\infty} \frac{(-1)^{k+1} x^{6k+3}}{(2k+1)!}
\]

(b) \[
\sum_{k=1}^{\infty} 2^k \cdot k \cdot x^{k-1}
\]
You may use any of the following for the rest of the exam.

\[
\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^k + \cdots = \sum_{k=0}^{\infty} x^k, \quad \text{for } |x| < 1
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\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + \frac{x^k}{k} + \cdots = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}x^k}{k}, \quad \text{for } -1 < x \leq 1
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e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^k}{k!} + \cdots = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \quad \text{for } |x| < \infty
\]

\[
\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{(-1)^{k+1}x^{2k+1}}{(2k+1)!} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, \quad \text{for } |x| < \infty
\]

\[
\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{(-1)^k x^{2k}}{(2k)!} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}, \quad \text{for } |x| < \infty
\]

12. (8 points) Use a Taylor series to approximate the following definite integral. Retain as many terms as needed to ensure the error is less than \(\frac{1}{5000}\). Justify and simplify your answer.

\[
\int_{0}^{0.1} \frac{\ln(1 + x)}{x} \, dx
\]
Name ________________________________

Circle your section:

32-01 Hao Liang TThF 8:30-9:20

32-02 Mary Glaser TThF 12-12:50 SECRET 4 digit code number: ________

32-03 Gail Kaufmann TTHF 12-12:50

32-04 Thomas Benson TTh1:30-2:20, F2:30-3:20

I pledge that I have neither given nor received assistance on this exam.

Signature ________________________________