1. (18 points) Integrate or evaluate. Simplify your answer in part (b).

(a) \( \int \sin^2\left(\frac{x}{2}\right) \, dx \)  
(b) \( \int_0^1 \frac{x + 7}{x^2 - x - 2} \, dx \)  
(c) \( \int \sqrt{x \ln x} \, dx \)

2. (18 points) Determine whether each of the following series converges or diverges. Justify your answer. State and check hypotheses of any test, rules or theorems you use. You may not simply quote a theorem.

(a) \( \sum_{n=2}^{\infty} \frac{1}{n \ln n} \)  
(b) \( \sum_{n=1}^{\infty} \frac{3n^2}{8^n} \)  
(c) \( \sum_{n=1}^{\infty} \frac{(-1)^n n^2}{3n^8 + 10} \)

3. (8 points) Find the radius of convergence and interval of convergence for the following power series:

\[ \sum_{n=1}^{\infty} \frac{(-1)^n (x + 2)^n}{4^n \sqrt{n}} \]

4. (4 points) Sketch each of the following sets of points in the polar plane. (Use separate sketches for each set.)

(a) \( \{(r, \theta) : 1 \leq r < 2 \text{ and } -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}\} \)

(b) \( \{(r, \theta) : r \leq 0 \text{ and } \frac{\pi}{6} \leq \theta \leq \frac{\pi}{3}\} \)

5. (8 points) Compute the Taylor series for \( f(x) = xe^x \) centered at \( a = 1 \) using the definition of the Taylor series. Write the series using summation notation. Find the interval of convergence.

6. (6 points) Consider the following parametric equations:
\( x = 3 - 2\cos t \) and \( y = 4\sin^2 t ; \quad \pi \leq t \leq 2\pi \)

(a) Eliminate the parameter \( t \) to obtain an equation in \( x \) and \( y \).

(b) Sketch the curve and indicate the positive orientation.

please turn over
You may use any of the following for problems 7 and 8.

\[
\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^k + \cdots = \sum_{k=0}^{\infty} x^k, \quad \text{for } |x| < 1
\]

\[
e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^k}{k!} + \cdots = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \quad \text{for } |x| < \infty
\]

\[
\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{(-1)^k x^{2k+1}}{(2k+1)!} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, \quad \text{for } |x| < \infty
\]

\[
\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{(-1)^k x^{2k}}{(2k)!} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}, \quad \text{for } |x| < \infty
\]

7. (12 points) Find series representations and **radii of convergence** for each of the following functions. Write the series using summation notation.

(a) \( f(x) = \frac{x^4}{6 + x} \)

(b) \( f(x) = \ln(1 - 2x) \)

8. (4 points) Identify each of the functions represented by the following power series.

(a) \( \sum_{k=0}^{\infty} \frac{(-1)^{k+1} x^{4k}}{(2k)!} \)

(b) \( \sum_{k=0}^{\infty} \frac{(-1)^k x^{k+3}}{5k!} \)

9. (8 points) Polar Curves

(a) Sketch the cardioid \( r = 2 + 2\sin\theta \) and the circle \( r = 6\sin\theta \) in the same polar graph and shade the region outside the cardioid and inside the circle.

(b) Set up but **do not evaluate** a definite integral that expresses the area of the region described in part(a).

10. (14 points) Taylor Polynomials and Remainder

(a) Let \( f(x) = \ln(x) \).

i. Find an expression for the remainder term \( R_2(x) \) in the second order Taylor polynomial for \( f(x) = \ln(x) \) centered at \( a = 1 \).

ii. Use this remainder term to estimate the absolute error in approximating \( \ln(1/2) \) with \( p_2(1/2) \).

(b) Find the third Taylor polynomial \( p_3(x) \) for the function \( f(x) = \sin(2x) \) centered at \( a = \pi/3 \). Evaluate all coefficients.

End of exam. Have a great holiday.