Math 21 Section 2

Review 2

1. One day when looking in your garage you find a box of eighty old landline telephones. You decide to send an email to an email list consisting of 150 people, asking if any of them have landline service, with the hope that they will take your old telephones. A recent study found that two out of every five households in the U.S. do not have landline service. Let $x$ be the random variable that counts the number of people on the email list who have landline service.

(a) Write an expression for the probability that exactly 80 people on the email list have landline service.

(b) Under what condition can you approximate $x$ with a normal distribution?

   Check that those conditions are satisfied.

(c) Use the normal distribution to approximate the probability that exactly 80 people have landline service.

(d) Use the normal distribution to approximate the probability that at least 80 people have landline service.

   (a) $P(x = 80) = \binom{150}{80} \cdot 0.6^{80} \cdot 0.4^{70}$

   (b) $np \geq 5$ and $nq \geq 5$. In this case $np = 90$ and $nq = 60$, so the conditions are satisfied.

   (c) $P(x = 80) \approx P\left(\frac{79.5 - np}{\sqrt{npq}} \leq z \leq \frac{80.5 - np}{\sqrt{npq}}\right) = P\left(-1.75 \leq z \leq -1.58\right) = P(z \leq -1.58) - P(z \leq -1.75) = 0.0571 - 0.0401 = 0.0170$

   (d) $P(x \geq 80) \approx P\left(z \geq \frac{79.5 - np}{\sqrt{npq}}\right) = P(z \geq -1.75) = 0.9599$

2. A plane filled with students coming home from spring break is about to begin boarding. The plane’s overhead storage bins can handle at most 2,450 lbs., and are being filled with suitcases that have normally distributed weights with a mean of 48 lbs. and standard deviation of 14 lbs.. If the probability of going over that limit is larger than .05, the airline will put all the bags in the cargo hold.

(a) If 49 passengers have one bag each, what is the maximum mean weight of a bag?

(b) What is the probability of a single bag exceeding the maximum mean weight?

(c) What is the probability that the load of 49 bags will exceed the load limit of the plane?

(d) Should the airline start putting bags in the cargo hold?

   (a) $2450/49 = 350/7 = 50$ is the maximum mean safe weight.

   (b) For this individual data value, $z = (50 - 48)/14 = 1/7 = 0.143$, so the probability is $1 - .5557 = .4443$.

   (c) The Z-score for a sample of 49 bags is $(50 - 48)/(14/\sqrt{49}) = 2/2 = 1$, so the probability is $1 - .8413 = .1587$.

   (d) .1587 is higher than 0.05, so the airline should put their bags in the cargo hold, and not in the overhead bins.

3. An advertising agency wishes to know what proportion of children prefer summer over all other seasons. To answer this, 100 children are surveyed, and 64 say that summer is their favorite season.

(a) Construct a 95% confidence interval for the proportion of children who like summer best.
(b) Test the hypothesis that more than half of all children prefer summer ($H_1 : p > 0.5$) with 95% confidence.

(a) The confidence interval is $\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \frac{64}{100} \pm 1.96 \sqrt{\frac{\frac{64}{100} \cdot \frac{36}{100}}{100}} = 0.64 \pm 1.96 \cdot \frac{8}{10} \cdot \frac{6}{10} = 0.64 \pm 1.96(0.048) = 0.64 \pm 0.094$$

Thus the confidence interval is $0.546 < p < 0.734$

(b) The null hypothesis is $H_0 : p = 0.5$. Because our hypothesis test is one-sided, the critical value is $1.645$. The test statistic is

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{\frac{64}{100} - \frac{50}{100}}{\sqrt{\frac{\frac{50}{100} \cdot \frac{50}{100}}{100}}} = \frac{14}{5} = 2.8.$$ 

$2.8 > 1.645$, so we reject the null hypothesis.

4. A news station would like to be the first to report the results of a local election without having to wait until the full results are available. To do so, the station takes an exit poll of 2,500 voters and observes that 1625 of them have just voted for Frank Underwood. They wish to test the hypothesis that the proportion of voters who voted for the candidate is greater than half ($H_0 : p = 0.5, H_1 : p > 0.5$) with 95% confidence (that is, a significance level of $\alpha = 0.05$) so that they may call the race for Frank Underwood.

(a) What is your sampling distribution?
(b) What is your critical value?
(c) What is your test statistic?
(d) What is your p-value?
(e) What is your conclusion?

(a) Since this is testing a claim about proportions, the sampling distribution is the normal distribution.
(b) The critical value is 2.325.
(c) The test statistic is

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{1625/2500 - 1250/2500}{\sqrt{(0.5)(0.5)/2500}} = \frac{375/2500}{.01} = \frac{15/100}{1/100} = 15.$$
(d) The p-value is smaller than .0001
(e) The test statistic is in our critical region (15 > 2.325) and the p-value is smaller than \( \alpha \) (0.05), so we reject the null hypothesis. The station should call the race for Mr. Underwood.

![Normal distribution curve with critical regions](image)

5. A commonly quoted factoid is that the average person sleeps 8 hours a day. To test this claim, a lab enlists 36 volunteers for an overnight study, and measures that the number of hours each person slept. Their observations are recorded as \( \bar{x} = 7.4 \) hours and \( s = 1.2 \) hours. They wish to test the hypothesis that the mean number of hours a person sleeps is eight. \( (H_0 : \mu = 8, H_1 : \mu \neq 8) \) with 99% confidence (that is a significance level of \( \alpha = .01 \)).

(a) What is your sampling distribution?
(b) What are your critical values?
(c) What is your test statistic?
(d) What is your conclusion?

(a) Since we do not know \( \sigma \), we must use a \( t \) distribution.
(b) The critical values are \( \pm 2.724 \).
(c) The test statistic is
\[
t = \frac{7.4 - 8}{1.2 / \sqrt{36}} = \frac{-0.6}{0.2} = -3
\]
(d) The test statistic is in our critical region \((-3 < -2.724)\), so we reject the null hypothesis.

![Normal distribution curve with critical regions](image)

6. An advertising agency wishes to know the average number of text messages a typical student sends in one month. They take a simple random sample of 25 students and have them give the number of sent texts counted in the previous month’s bill. The sample has a mean of \( \bar{x} = 1800 \) texts and standard deviation of \( s = 500 \) texts.
(a) What is the sample distribution?
(b) How many degrees of freedom are there?
(c) Estimate the mean number of texts a typical student sends using a 95% confidence interval.

(a) Since we do not know $\sigma$, we must use a $t$ distribution.
(b) $n-1 = 25 - 1 = 24$
(c) The confidence interval is
\[
\bar{x} - t_{\alpha/2} \frac{s}{\sqrt{n}} < \mu < \bar{x} - t_{\alpha/2} \frac{s}{\sqrt{n}}
\]
\[
1800 - 2.064 \frac{500}{5} < \mu < 1800 + 2.064 \frac{500}{5}
\]
\[
1800 - 206.4 < \mu < 1800 + 206.4
\]
\[
1599.6 < \mu < 2006.4
\]

7. For quality control purposes, a chocolatier wishes to estimate how much the weights of their king-sized chocolate bars vary. To do so, they take a random sample of 13 bars, and find the sample standard variance to be .1 ounces.

(a) What is your sample distribution?
(b) How many degrees of freedom are there?
(c) Find the left and right critical values for $\alpha = 0.05$.
(d) Write an expression for a 95% confidence interval to estimate variance.
(e) Write an expression for a 95% confidence interval to estimate standard deviation.

(a) We use a $\chi^2$ distribution.
(b) $df = n - 1 = 12$
(c) We need 0.975 and 0.025 on the right. Chart A-4 gives $\chi^2_L = 4.404$ and $\chi^2_R = 23.337$
(d) The confidence interval is
\[
\frac{(n-1)s^2}{\chi^2_R} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_L}
\]
\[
\frac{(12)(0.1)}{23.337} < \sigma^2 < \frac{(12)(0.1)}{4.404}
\]
\[
(.0514 < \sigma^2 < .2725)
\]
(e) The confidence interval is
\[
\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}
\]
\[
\sqrt{\frac{(12)(0.1)}{23.337}} < \sigma^2 < \sqrt{\frac{(12)(0.1)}{4.404}}
\]
\[
(.0514 < \sigma < .2725)
\]
\[
(.2267 < \sigma < .5220)
\]
8. A manufacturer of woven scarves claims that the variance in the lengths of their scarves is 1 inch. To test this claim, a quality control expert buys 11 scarves and finds the sample standard deviation to be .9 inches. The expert wishes to test the hypothesis that the variance is smaller than claimed ($H_0 : \sigma^2 = 1, H_1 : \sigma^2 < .1$) with 99% confidence (that is a significance level of $\alpha = .01$).

(a) What is the sample distribution?
(b) How many degrees of freedom are there?
(c) Find the critical value(s).
(d) Find the test statistic.
(e) What is your conclusion?

(a) We use a $\chi^2$ distribution.
(b) $df = n - 1 = 10$
(c) This is a left-tail test with $\alpha = .01$ and 10 degrees of freedom. Chart A-4 gives areas to the right, so we get a critical value of $\chi^2 = 2.558$
(d) The test statistic is $\chi^2 = \frac{(10)(.9)^2}{1^2} = 10(.81) = 8.1$
(e) The critical region is to the left of 2.558. Our test statistic does not fall in the critical region. We cannot reject the null. The manufacturer’s claim seems to be valid.