Nassau County, NY Board of Supervisors

A: Hempstead 1  B: Hempstead 2  C: North Hempstead
D: Oyster Bay  E: Long Beach  F: Glen Cove

In the 1960's

[58: 31, 31, 28, 21, 2, 2] , which is equivalent to the system [ 16: 9, 9, 7, 3, 1, 1]

1. Who are the dummies in this system?

2. Use the system [ 16: 9, 9, 7, 3, 1, 1] (and the identification of the dummies in question 1) to find the Banzhaf power distribution for this system.

3. Find the minimal equivalent system.

In the 1990's

[65: 30, 28, 22, 15, 7, 6] , which is equivalent to the minimal system [ 15: 7, 6, 5, 4, 2, 1]

4. Are there any 2-player winning coalitions?
Still using \([15: 7, 6, 5, 4, 2, 1]\)

5. List all the 3-player winning coalitions, and count the number of times each player is critical.

6. Here are all the 4-player winning coalitions. Circle the critical players.

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<th>A</th>
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</table>

7. List all the 5-player winning coalitions. Circle the critical players.

8. Is anyone critical in the grand coalition?

9. Find the Banzhaf power distribution.

10. Compare the power distribution to the one from the 1960's.
In the 1960's

[58: 31, 31, 28, 21, 2, 2], which is equivalent to the system [16: 9, 9, 7, 3, 1, 1]

1. Who are the dummies in this system?
   \[D, E, F\] (They have \(3+1+1<5\), which is never helpful to \(A, B, C\))

2. Use the system [16: 9, 9, 7, 3, 1, 1] (and the identification of the dummies in question 1) to find the Banzhaf power distribution for this system.

\[
\begin{pmatrix}
16 & 9 & 9 & 7 & 3 & 1 & 1 \\
\end{pmatrix}
\]

\[
A \quad B \quad C \\
A \quad B \quad C
\]

\[
\begin{align*}
\beta_A &= \beta_B = \beta_C = \frac{1}{3} \\
\beta_D &= \beta_E = \beta_F = 0
\end{align*}
\]

3. Find the minimal equivalent system.

\[
\begin{pmatrix}
2 & 1 & 1 & 0 & 0 & 0 \\
\end{pmatrix}
\]

In the 1990's

[65: 30, 28, 22, 15, 7, 6], which is equivalent to the minimal system [15: 7, 6, 5, 4, 2, 1]

4. Are there any 2-player winning coalitions? \textbf{NO.}
Still using $[15: 7, 6, 5, 4, 2, 1]$

5. List all the 3-player winning coalitions, and count the number of times each player is critical.

\[
\begin{array}{c}
A \ B \ C \\
A \ B \ D \\
A \ B \ E \\
A \ C \ D \\
B \ C \ D \\
\end{array}
\Rightarrow all \ critical
\begin{array}{c}
A - 4 \\
B - 4 \\
C - 3 \\
D - 3 \\
E - 1 \\
\end{array}
\]

6. Here are all the 4-player winning coalitions. Circle the critical players.

\[
\begin{array}{c}
A \ B \ C \ D \\
A \ B \ C \ E \\
A \ B \ C \ F \\
A \ B \ D \ E \\
A \ B \ D \ F \\
A \ B \ C \ D \ F \\
A \ B \ C \ E \ F \\
A \ B \ C \ D \ E \ F
\end{array}
\Rightarrow \begin{array}{c}
A - 8 \\
B - 7 \\
C - 6 \\
D - 5 \\
E - 2 \\
F - 1 \\
\end{array}
\]

7. List all the 5-player winning coalitions. Circle the critical players.

\[
\begin{array}{c}
A \ B \ C \ D \ E \\
A \ B \ C \ D \ F \\
A \ B \ C \ E \ F \\
A \ B \ D \ E \ F \\
A \ B \ C \ D \ E \ F
\end{array}
\Rightarrow \begin{array}{c}
A - 3 \\
B - 2 \\
C - 2 \\
D - 1 \\
\end{array}
\]

8. Is anyone critical in the grand coalition? \textbf{No}

\[
ABCDEF
\]

9. Find the Banzhaf power distribution.

\[
\beta_A = \frac{15}{52}, \ \beta_B = \frac{13}{52}, \ \beta_C = \frac{11}{52}, \ \beta_D = \frac{9}{52}, \ \beta_E = \frac{3}{52}, \ \beta_F = \frac{1}{52}
\]

10. Compare the power distribution to the one from the 1960's.

\[
\frac{1}{3} = \frac{17.3}{52}
\]

so A, B, C each lost some power

D, E, F all gained.