1. Use the following preference schedule to show elimination and Coombs can result in the same winner, while the runoff winner will not be the same.

<table>
<thead>
<tr>
<th>3</th>
<th>2</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>2</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>B</td>
<td>D</td>
<td>D</td>
<td>E</td>
<td>B</td>
<td>B</td>
<td>E</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>C</td>
<td>A</td>
<td>C</td>
<td>A</td>
<td>A</td>
<td>C</td>
<td>E</td>
<td>E</td>
<td>C</td>
</tr>
<tr>
<td>D</td>
<td>D</td>
<td>C</td>
<td>A</td>
<td>B</td>
<td>E</td>
<td>D</td>
<td>C</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>E</td>
<td>E</td>
<td>E</td>
<td>E</td>
<td>E</td>
<td>B</td>
<td>B</td>
<td>A</td>
<td>D</td>
<td>B</td>
</tr>
<tr>
<td>B</td>
<td>A</td>
<td>D</td>
<td>D</td>
<td>C</td>
<td>C</td>
<td>A</td>
<td>D</td>
<td>A</td>
<td>D</td>
</tr>
</tbody>
</table>

2. Use the following preference schedule to show elimination and Coombs can result in different winners.

<table>
<thead>
<tr>
<th>5</th>
<th>2</th>
<th>1</th>
<th>4</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>F</td>
<td>S</td>
<td>S</td>
<td>D</td>
</tr>
<tr>
<td>X</td>
<td>S</td>
<td>X</td>
<td>D</td>
<td>X</td>
</tr>
<tr>
<td>S</td>
<td>X</td>
<td>D</td>
<td>X</td>
<td>S</td>
</tr>
<tr>
<td>D</td>
<td>D</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

3. Use the preference schedule from problem 1 to show:

(a) Plurality is not Condorcet-fair.
(b) Plurality is not Smith-fair.
(c) Runoff is not Condorcet-fair.
(d) Runoff is not Smith-fair.
(e) Runoff is not monotonic.

4. Consider the preference schedule below:

(a) Show elimination is not monotonic.
(b) Find the winner using beatpath.

<table>
<thead>
<tr>
<th>12</th>
<th>10</th>
<th>7</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>K</td>
<td>B</td>
<td>H</td>
</tr>
<tr>
<td>B</td>
<td>T</td>
<td>H</td>
<td>B</td>
</tr>
<tr>
<td>K</td>
<td>H</td>
<td>K</td>
<td>T</td>
</tr>
<tr>
<td>H</td>
<td>B</td>
<td>T</td>
<td>K</td>
</tr>
</tbody>
</table>
5. Use the preference schedule from problem 2 to show that Borda is not Smith-fair.

6. Use the following preference schedule to show that plurality allows both losing spoilers and weak spoilers.

   | 4 3 3 2 |
   | A B C D |
   | B C A C |
   | C D D B |
   | D A B A |

7. Consider the following preference schedule (from Mathematical Excursions, p. 18):

   | 2 6 4 1 1 4 4 |
   | A B B C C D E |
   | D A A B D A C |
   | C C D A A E D |
   | B D E D B C B |
   | E E C E E B A |

With the following one-to-one comparisons:

   A : B 7 : 15
   A : C 16 : 6
   A : D 13 : 9
   A : E 18 : 4
   B : C 10 : 12
   B : D 11 : 11
   B : E 14 : 8
   B : E 18 : 4
   C : D 12 : 10
   C : E 10 : 12
   D : E 18 : 4

(a) Show pairwise comparison allows losing spoilers.
(b) Find the beatpath winner(s).

8. Consider the following preference schedule:

   | 12 6 4 4 3 4 |
   | T K K B H H |
   | B T T H K B |
   | K H B K T T |
   | H B H T B K |

With the following one-to-one comparisons:

   T : B 25 : 8
   T : H 22 : 11
   B : H 20 : 13
   T : K 16 : 17
   B : K 20 : 13
   K : H 22 : 11

(a) Show the plurality winner is different than the a priori Smith fair plurality winner.
(b) Show the elimination winner is different than the a priori Smith fair elimination winner.
9. Consider the following preference schedule (from Mathematical Excursions, p. 13):

<table>
<thead>
<tr>
<th>93  44  10  30  42  81</th>
</tr>
</thead>
<tbody>
<tr>
<td>A B C C D E</td>
</tr>
<tr>
<td>B D A E C D</td>
</tr>
<tr>
<td>C E E B E C</td>
</tr>
<tr>
<td>D C B A A B</td>
</tr>
<tr>
<td>E A D D B A</td>
</tr>
</tbody>
</table>

With the following one-to-one comparisons:

\[
\begin{align*}
A : B & : 145 : 155 \\
A : D & : 133 : 167 \\
B : C & : 137 : 163 \\
B : E & : 137 : 163 \\
C : E & : 175 : 125 \\
A : C & : 93 : 207 \\
A : E & : 103 : 197 \\
B : D & : 177 : 123 \\
C : D & : 133 : 167 \\
D : E & : 179 : 121
\end{align*}
\]

Find the winner(s) using:

(a) Plurality
(b) a priori Smith fair plurality
(c) a posteriori Smith fair plurality
(d) elimination
(e) pairwise
(f) beatpath

10. For the preference schedule in problem 2, find the one-shot and recursive rankings for plurality, Borda, pairwise comparison, and elimination.

11. Consider the following preference schedule:

<table>
<thead>
<tr>
<th>5  2  2  4  3  4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A A B B C D</td>
</tr>
<tr>
<td>B C D C D B</td>
</tr>
<tr>
<td>C D C D B A</td>
</tr>
<tr>
<td>D B A A A C</td>
</tr>
</tbody>
</table>

With the following one-to-one comparisons:

\[
\begin{align*}
A : B & : 7 : 13 \\
A : D & : 7 : 13 \\
B : D & : 11 : 9 \\
A : C & : 11 : 9 \\
B : C & : 15 : 5 \\
C : D & : 14 : 6
\end{align*}
\]

(a) Draw the graph with margins, and find the Smith set.
(b) Find the winner(s) using runoff and beatpath.
(c) Find the 1-shot ranking using plurality.
(d) Find the 1-shot ranking using Coombs.
(e) Find the 1-shot ranking using elimination.
(f) Find the 1-shot ranking using Borda.
(g) Find the 1-shot ranking using pairwise comparison.
(h) Find the recursive ranking using plurality.
(i) Find the recursive ranking using Coombs.
(j) Find the recursive ranking using elimination.
(k) Find the recursive ranking using Borda.
(l) Find the recursive ranking using pairwise comparison.
(m) What is the set of winners for any Smith fair method?
(n) Show that C is a losing, weak spoiler using plurality or runoff.
(o) Show that B is a losing spoiler using plurality.
(p) Show that C is not a spoiler using Borda.
(q) Show that elimination violates monotonicity in this election (we did this in class).

12. Consider the following preference schedule:

<table>
<thead>
<tr>
<th></th>
<th>5</th>
<th>4</th>
<th>2</th>
<th>2</th>
<th>3</th>
<th>1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>D</td>
<td>E</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>D</td>
<td>E</td>
<td>A</td>
<td>E</td>
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<td></td>
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<td>D</td>
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<tr>
<td>E</td>
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<td>A</td>
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<td>C</td>
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<tr>
<td>D</td>
<td>E</td>
<td>C</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td></td>
</tr>
</tbody>
</table>

With the following one-to-one comparisons:

<table>
<thead>
<tr>
<th></th>
<th>13:7</th>
<th>7:13</th>
<th>6:14</th>
<th>11:9</th>
<th>11:9</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>E</td>
</tr>
<tr>
<td>A</td>
<td>C</td>
<td>16:4</td>
<td>12:8</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>A</td>
<td>C</td>
<td>16:4</td>
<td>12:8</td>
<td>11:9</td>
<td>11:9</td>
</tr>
<tr>
<td>A</td>
<td>C</td>
<td>16:4</td>
<td>12:8</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>A</td>
<td>C</td>
<td>16:4</td>
<td>12:8</td>
<td>D</td>
<td>E</td>
</tr>
<tr>
<td>A</td>
<td>C</td>
<td>16:4</td>
<td>12:8</td>
<td>D</td>
<td>E</td>
</tr>
<tr>
<td>A</td>
<td>C</td>
<td>16:4</td>
<td>12:8</td>
<td>D</td>
<td>E</td>
</tr>
</tbody>
</table>

(a) Draw the graph with margins, find the sequential winner, and the Smith set.
(b) Find the winner(s) using runoff and beatpath.
(c) Find the 1-shot ranking using plurality.
(d) Find the 1-shot ranking using Coombs.
(e) Find the 1-shot ranking using elimination.
(f) Find the 1-shot ranking using Borda.
(g) Find the 1-shot ranking using pairwise comparison.
(h) Find the recursive ranking using plurality.
(i) Find the recursive ranking using Coombs.
(j) Find the recursive ranking using elimination.
(k) Find the recursive ranking using Borda.
(l) Find the recursive ranking using pairwise comparison.
(m) Is B a polarizing candidate in this election?
(n) Show that elimination violates monotonicity in this election.
(o) Why are there no weak spoilers in this election, no matter what voting method is used?
(p) Show that A is a losing spoiler using plurality.
(q) Show that B and E are both losing spoilers using pairwise comparison.
13. Consider this graph of a preference schedule:

Note that the margin from A to E is 3, the margin from C to A is 3, the margin from D to A is 7, and the margin from E to C is 5, and all the other margins are 1.

(a) Find the sequential comparison winner, assuming alphabetical order. Show some work.

(b) Find the Smith Set.

(c) Find the beatpath winner(s). Show disqualifications with strengths as we have in class.

(d) Find $D_B$.

(e) Find $D_F$.

(f) Find the strength of the strongest beatpath from C to A.

(g) Find the strength of the strongest beatpath from A to D.

(h) Find the strength of the strongest beatpath from D to E.

14. Consider the following preference schedule:

<table>
<thead>
<tr>
<th>2 6 6 1 4 4 1 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>C C A C E D D D</td>
</tr>
<tr>
<td>A B E E C A C A</td>
</tr>
<tr>
<td>B A D D D C A E</td>
</tr>
<tr>
<td>D D B B B E E B</td>
</tr>
<tr>
<td>E E C A A B B C</td>
</tr>
</tbody>
</table>

With the following one-to-one comparisons:

\[
\begin{align*}
A : B & \ 15 : 11 \\ 
A : D & \ 14 : 12 \\ 
B : C & \ 8 : 18 \\ 
B : E & \ 8 : 18 \\ 
C : E & \ 14 : 12 \\
A : C & \ 12 : 14 \\ 
A : E & \ 21 : 5 \\ 
B : D & \ 8 : 18 \\ 
C : D & \ 13 : 13 \\ 
D : E & \ 15 : 11 \\
\end{align*}
\]

(a) Draw the graph.

(b) Find the sequential comparison winner, assuming alphabetical order. Show some work.

(c) Find the Smith Set.
(d) Find the plurality winner(s)
(e) Find the runoff winner(s).
(f) Find the elimination winner(s).
(g) Find the pairwise comparison winner(s).
(h) Find the Borda winner(s).
(i) Find the Coombs winner(s).
(j) Find the a priori Smith fair Borda winner(s).
(k) Show that A is a losing spoiler using Pairwise comparison.
(l) Show that A is a losing spoiler using Plurality.
(m) Show that A is a losing spoiler using beatpath.
(n) Show that C is a winning spoiler using elimination.